A HIRE, Phillipe de

GNOMONICKS,

Pendock OR, *Neale*

The Art of Shadows IMPROVED.

Plainly set forth in the Drawing of

SUN-DIALS

On all sorts of

PLANES

By Different Methods.

With the Geometrical Demonstrations of all the Operations.

By Mr. *DELAHIRE* of the Royal Academy of Sciences in *PARIS*.

Englished and Illustrated with Cutts.

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THE
PREFACE.

I Have always confidered the Defcription of *Sun-Dials* as one of the moft ingenious and ufeful Inventions derived from the Study of the *Mathematicks*. Alfo there is nothing that draws more Admiration from all Men, than to fee Strait-lines drawn on a *Plane* at *Unequal Diftances*, to meafure exactly the equal Divifions of the time of the Continuance of a Day: and altho the *Sun* appears in different places of Heaven according to the different Seafons of the Year, yet the fame Strait-lines do ftill determine the fame hour at all thefe different Seafons.

But thefe Hour-lines have nothing in them that deferves to be confidered, if we compare them to thofe which trace forth the way which the *Sun* makes in his Diftances from the Equinoctial Line, which queftionlefs have given place to the more profound Meditati-

tions of the *Sections* of a *Cone* ; which at this day is the moſt conſiderable part of our Speculative *Geometry.*

I might eaſily demonſtrate that we are beholding to *Sun-Dials* for the diſcovery of thoſe admirable Curve Lines whereof we find very great uſe in all parts of the *Mathematicks* ; for we cannot conſider the Shadow of the end of any body pointed on a *Plane*, without perceiving at the ſame time the Curvature which the way of the *Sun* traces forth ; which is moſt like to that of the *Section* of an upright *Cone* which hath a Circle parallel to the Equinoctial for its Baſe , on which we may ſuppoſe the *sun* moves then when he makes that Shadow.

But altho the Properties of theſe Curve Lines ſerves as a Foundation for the moſt part of the Deſcriptions of *Dials* , yet in this Work I do not intend to explain any thing thereof in particular, for that would be to depart too much from my Subject.

Seeing alſo that divers other knowing *Geometers* have largely treated there-

thereof, it ſeems to me that it will be very uſeleſs to re-ſearch into the time of the moſt ancient Antiquity, and who was the Inventer of this Art: We may only believe very likely that it was perfected by little and little, and that the firſt Men ſeeing the neceſſity which they had to divide the continuance of a day into divers parts, judged that it could not be better done than by the ſame *Sun* which limits the continuance thereof.

We may alſo be very eaſily perſwaded that the Meridian Line hath been the firſt which they have drawn, as well becauſe that it divides the apparent day, which is the time during which we may ſee the *Sun*, into two equal parts, and that the Shadow of a Strait-line raiſed perpendicularly upon an Horizontal Plane, was always extended along this Meridian Line when the top of it meets with it, although it be in different places; ſo that it ſerves us to know the greateſt height of the *Sun* above the Horizon every day, the which changes during the time of half a Year,

it ſeemed to them one of the moſt conſiderable Phenomena's of the *Sun.*

The moſt part of the Ancients divided the ſpace of time (which is from the riſing to the going down of the Sun) into twelve equal parts, which they called Hours, and they began their Account from Sun riſing; and although the Hours changed their length during the half year to thoſe which did not Inhabit under the Eqninoctial Line, where the apparent days are always equal to one another, therefore they have always Mid-day at the Sixth Hour: but theſe ſorts of Hours have no Circles of the Sphere that repreſent them.

The *Babylonians* began the day at the Sun Riſing, and divided the Duration into 24 equal Hours: The *Italians* began it at Sun Setting, and made alſo their Hours equal: Theſe two manners of beginning the day have the Horizon for their term; the firſt did always know how long time they had from the Sun Riſing above the Horizon, and the others did always know that which remains to the Sun Setting.

The Aſtronomers and the greateſt part of the Nations of *Europe* begin their Day when the Sun comes to the Meridian,theſe laſt when the Sun comes to the Meridian under the Horizon ; and the firſt when the Sun comes to the Meridian above the Horizon ; this manner of beginning the day has great advantages beyond other.

I undertake in this Work only the Deſcription of the Aſtronomick Hours , which have for their Term the Meridian , and of the *Italians* and *Babylonians* that begin from the Horizon. In the Conſtruction of theſe ſorts of *Dials* there are two principal Operations which we may conſider each in particular , which has obliged me to divide this Treatiſe into Two Parts.

In the Firſt Part , after I have explained as brief as I can , that which is neceſſary to be known for the right underſtanding of the Conſtruction of *Sun Dials* , in which I give the definition with that of all its parts : I propoſe following divers

Manners and Practices, to draw their principal Line, with the Points which are neceſſary for the deſcription of the Hour Lines, to the intent that we may ſerve our ſelves with thoſe which are moſt fit in the different Rencontres of the *Planes* propoſed.

Each of theſe Practices have advantages in particular Caſes; I obſerve by which we may ſerve our ſelves very near following the Expoſition of the *Plane* propoſed, to the intent that thoſe that have not ſufficient knowledge of the different Rencontres of *Planes* with the Circles of the Sphere, may not give themſelves the trouble to follow a Method from which they cannot draw a great advantage.

In all theſe Practices which I propoſe, I make no uſe of the Magnetical Needle, for there happens great change to the Variation of the Magnetical Needle; beſides we are not aſſured that there is no Iron hid, or ſome Stone or Brick which is of the Nature of Iron, which may turn aſide the

Needle

Needle from its true Direction towards the place whither it would go if it were free.

Alſo I approve not that Method which many do practice to find the Declination of a *Plane*, that is to ſay, the Angle which the Meridian Line makes on a Horizontal Plane with the Horizontal Line, which is the meeting of the Plane of the *Dial* with that Plane: they draw on a Plane ſet Parallel to the Horizon (the which we call a Level Plane) a Meridian Line following one of the Practices, which may be ſeen hereafter; and when the Sun marks Mid-day on that Horizontal Plane, they mark a Point of Shadow on the propoſed Plane; but you muſt obſerve that the Errors that one commits in all the Operations, as well in the placing of the Plane level, as in the determination of the Meridian, are multiplied and increaſed in tranſporting them to another *Plane*.

For the ſame Reaſons we ought alſo to reject all ſorts of Inſtruments, unleſs they be very plain and very large: There-

Therefore I have thought good to use only the Ruler and Compass, the Plomb Line and the Level, and to draw the Lines and Circles only upon the given *Plane*.

Altho we may determine the length of the Lines, and also the most part of the Angles by Calculation of Spherical and Strait-lin'd Triangles, which serves for the drawing of *Dials*, Yet I have thought good that I ought not to propose any (of those ways) in this small Treatise, because that the most part of those Calculations are much longer than the Practice, and they are founded but upon the same Angles, and the same Lines which I have used in the Practice; so that those that can use Calculation, shall find no great difficulty to apply them to Numbers and Sines, where I only propose Lines and Angles.

After I have taught to Draw the Principal Line of *Dials*, I proceed to the Second Part, wherein I shew to draw

draw the Aſtronomical Hour Lines, and then to deſcribe the Parallels of the Signs.

I propound no particular Conſtruction on the Horizontal and Vertical Planes, which only gives particular Rules for each Caſe, the which in the ordinary way happens very ſeldom: Therefore the Methods that I teach are for all ſorts of Planes indifferently conſidered. I know well that there are divers Caſes where we might find Abridgments, but theſe Abridgments conſiſts only in certain Lines and Points which come to be united in the general Practices which I give: Alſo it is to be obſerved that the Portions of Curve Lines that I deſcribe, are always Conick Sections, that is to ſay, either Ellipſes, Hyperboles, or Paraboles, and ſometimes Circles, when the Plane is perpendicular to the Axis of the Cone, the which is always an upright Cone.

The moſt part of the Practice that I teach being founded on the Declination of the Sun, I give you a Table

ble thereof Calculated for the days of Four Years following one another, to the end to comprehend the Biſſextile or Leap Year ; I there alſo add the Differences of the Declinations for every day, with a Table of Latitudes of the Principal Cities of the World.

THE

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THE FIRST PART.

CHAP. I.

Of the Circles *of the* Sphere *necessary to be known for the Drawing of* SUN-DIALS.

THE *Sphere* is an Instrument whereby we explain the *Daily Motion* of the *Cœlestial Bodies*, according as they appear to us to move always from *East* to *West*, and also the proper *Motion* of the *Sun*, which moves

from *West* toward the *East*, and makes his *Revolution* thro the *Twelve Cœlestial Signs* in the ſpace of one year.

This *Inſtrument* is compoſed of divers *Circles*, of which we only deſcribe thoſe that belong to our preſent *Subject* : Thoſe *Circles* whoſe *Planes* paſs thro the Center of the *Earth*, are called *Great Circles* of the *Sphere* , and all the other are *Leſs*.

But before we ſpeak of theſe *Circles*, we ought to conſider the *Axis* of the *Sphere*, which we conceive to be as a ſtrait line about which the *Inſtrument* is turned. The two Ends of the *Axis* are called *Poles* , the one *North* and the other *South*.

The *Earth* is placed in the middle of this *Inſtrument*, and conſequently the *Axis* paſſes thro the Center thereof. We may underſtand from *Aſtronomical Obſervations* that the *Globe* of the *Earth* is ſo little , in reſpect to its diſtance from the *Sun* , ſo that we may conſider it as a *Point* , if we compare it to that diſtance.

The *Equinoctial* or *Equator* is a *Great Circle*, and one of the chiefeſt of the *Sphere* , the *Plane* whereof is at Right-angles to the *Axis*, it divides the *Sphere* into two equal parts , whereof one is called *Septentrional*, and the other *Meridional*.

Fig. 1. The *Ecliptick* is another *Great Circle* whoſe *Plane* makes an angle with the *Plane* of the *Equinoctial* of 23 degrees 30 minutes ; the *Sun* moves under this *Circle* , going from the *West* toward the *East* , and makes one intire Revolution

in

in 365 days and near 6 hours. The Inclinatio of this *Circle* to the *Equinoctial*, causes the different *Declinations* of the *Sun* in regard to the *Equinoctial*; it is divided into Twelve equal parts, which are called *Signs*: And we begin from the Interſection thereof with the *Equinoctial*, proceeding towards the *North*. The Names of the Twelve *Signs* are *Aries*, *Taurus*, *Gemini*, *Cancer*, *Leo*, *Virgo*, *Libra*, *Scorpio*. *Sagittarius*, *Capricornus*, *Aquarius*, *Piſces*. Their Characters are ♈, ♉, ♊, ♋, ♌, ♍, ♎, ♏, ♐, ♑, ♒, ♓.

The *Tropicks* are two Circles parallel to the *Equinoctial*, which touches the *Ecliptick* in the point of its greateſt diſtance from the *Equinoctial*; therefore theſe Circles are diſtant from the *Equinoctial* 23 deg. 30 min. on one ſide toward the *North*, and on the other ſide toward the *South*.

So that it is manifeſt, That when the *Sun* is in the common Interſection of the *Ecliptick* and *Equator*, the Motion of the *Sphere* about its *Axis*, which goes from *Eaſt* to *Weſt*, and is called the Motion of the *Primum Mobile*, makes him appear to us in the *Equinoctial*; and alſo when he is in his greateſt diſtance from the *Equinoctial*, the ſame Motion of the *Primum Mobile* makes him to appear to us to move in the *Tropicks*.

The *Zenith* is an imaginary Point in the *Sphere*, marked by a ſtrait Line coming from the Center of the *Earth*, and paſſing by ſome place of the Superficies thereof. This Line is called the *Vertical Line* of that place.

The *Horizon* is a Great Circle, whoſe Plane cuts the *Vertical Line* at Right-angles. The *Horizon* of a Place diſtinguiſhes the viſible part of Heaven of that Place, from that part of Heaven which is not there ſeen.

The *Meridian* is a Great Circle which paſſes thro the *Poles* and *Zenith*, the Plane whereof is at Right-angles with the Planes of the *Equinoctial* and *Horizon.* When the *Sun* comes to this Circle, he is in the middle of his apparent Courſe during a day, and is at his greateſt height above the *Horizon*, becauſe this Ci cle paſſes thro the *Zenith* and *Poles*.

If we ſuppoſe the *Equinoctial* to be divided into 24 equal parts, beginning from the *Meridian*, the 6th and 18th part ſhall fall on the Interſections of the *Horizon* and *Equinoctial*, becauſe the *Meridian* and *Horizon* are at Right-angles to one another; and if we imagine other Circles like the *Meridian*, that is to ſay, that paſs thro the *Poles* of the *World* and Point of Diviſion of the *Equinoctial*, thoſe Circles which we call *Meridians*, ſhall be the *Hour Circles*, among which is the *Meridian* of the Place, whereof all the Planes interſect one another in the *Axis*. We may alſo conceive others, which divide each part into two o four, to mark the half-hours and quarter-hours for if we ſuppoſe theſe Circles to be fix'd, the when the *Primum Mobile* turns the *Sun* with hi *Ecliptick* about the *Axis*, the time of his apparer Courſe ſhall be divided into hours, halves, an quarters, *by theſe Meridians.*

All

Alſo we number the *Declination* of the *Sun* upon the like *Meridians*, which do all interſect the *Equinoctial* at *Right Angles*, which we make to paſs thro the *Center* of the *Sun* in the *Ecliptick*: We number this *Declination* from the *Equinoctial* towards the *Poles*: Therefore it is either *South* or *North*. The *Angles* of *Declination* are meaſured by *Arches* of *Circles*.

Thoſe *Circles* that paſs by the *Vertical Line* are called *Vertical Circles* (or *Azimuth*) and their *Planes* are perpendicular to the *Plane* of the *Horizon*; they ſerve to meaſure the height of the *Sun* above the *Horizon*, which is numbred from the *Horizon* toward the *Zenith*.

It is manifeſt from that which has been ſaid before, that there are infinite *Horizons* and *Meridians*, and that there are only theſe two great *Circles*, which may change according to the different places on the *Earth*, for they are eſtabliſhed by the *Vertical Line*.

The *Amplitude* of *Riſing* or *Setting* is counted on the *Horizon*, beginning from the *Points* where the *Equinoctial* cuts the *Horizon*, and is numbred toward the *South* or *North*.

If we conceive that in the Revolution of one day the *Horizon* moves, as being faſtened to the *Axis*, ſo as it cannot change its Inclination, then when it ſhall paſs by the 24 equal Diviſions of the *Equinoctial*, it ſhall repreſent the 24 *Circles* of the *Italian* or *Babylonian* Hours.

CHAP. II.

Of the Definition of Sun-Dials, *and of the principal parts which serves for their Construction.*

THE Distance from the *Center* of the *Earth* to the *Superficies* thereof not being considerable, in respect of the Distance of the Earth from the *Sun*, we may take any *Point* on the *Superficies* and consider it as its *Center* in relation to the Motion of the *Sun*.

Therefore if we place a *Style* which is a *Pointed Rod* upon any *Plain Surface*, and then consider the *Point* of that *Style* as the *Center* of the *Earth*, the Intersections of that *Surface* with the *Planes* of the *Hour Circles*, of the *Equinoctial* or *Equator*, of the *Horizon* and of the other great *Circles*, shall be strait Lines, which retain the Names of the *Planes* of the *Circles* from whence they were produced: All these Lines on that *Plane Surface* with the *Style* makes a *Sun-Dial*.

The Shadow of the *Point* of the *Style*, which is one of the *Points* of the *Axis* shews the Hours.

And if the *Axis* which passes by the *Point* of that *Style* meets with the *Plane* of the *Dial* in any *Point*, that *Point* is called the *Center* of the

Dial;

Dial; for it is evident that all the *Hour Lines* ſhall meet in that *Point*.

It is alſo evident that the Shadow of the top of the *Style* gives the Hours, and ſhews when the *Sun* meets with any one of the *Circles* of the *Sphere*; for when the *Sun* comes to a great *Circle*, the Shadow of the *Axis* is extended in the *Plane* of that *Circle*, if that *Circle* paſſes by the *Axis*; and if it paſſes not by the *Axis*, the ſhadow of the *Point* of the *Style* ſhall be in the *Plane* of that *Circle*; for the *Planes* of great *Circles* paſs by the *Point* of the *Style*.

And if we conceive a *Conical Superficies* which has for its *Baſe* a leſs *Circle* of the *Sphere*, and for its *Vertex the Point of the Style*, that *Conical Superficies* ſhall meet the *Surface* of the *Dial* in a *Curve Line*; ſo as when the *Center* of the *Sun* ſhall touch that leſs *Circle* which is the *Baſe* of the *Conical Superficies*, the Shadow of the Point of the *Style* ſhall touch the *Curve Line* which is the meeting of that *Curve Superficies* with the *Plane* of the *Dial*: for the Point of the *Style* is on that *Superficies* whereof it is the *Vertex*.

The Foot of the *Style* is that Point on the *Plane* of the *Dial*, which is the meeting of a ſtrait Line drawn perpendicularly to that *Plane*, and which paſſes by the Point of the *Style*.

If the *Plane* of the *Dial* be conſidered as the *Plane* of the *Horizon* of any place, the ſtrait Line that paſſes by the Point of the *Style* and by its Foot, ſhall be the Vertical Line of that Place:

and the *Plane* that passes by that *Vertical* and by the *Axis*, shall be the proper *Meridian* of that Place, considered as the *Horizontal* of a Place.

The meeting of the *Meridian* and *Surface* of the *Dial* is called the *Substylar Line*, or the *Meridian* of the *Plane* or *Surface* of the *Dial*, which we ought to distinguish from the *Meridian* of the Place, which is the meeting of the *Meridian* proper to that Place and of the *Surface* of the *Dial*, at least if they be not coincident, which happens when the *Dial* does not decline from the East or West.

We see by the position of these Lines that the *Substylar Line* is always at *Right Angles* with the *Equinoctial Line*.

We ought to make the *Dial* so as the Foot of the *Style* be not incumbred, for that Point serves for many Operations; therefore the *Style* must be planted a little obliquely upon the *Surface*.

I understand by the length or height of the *Style* the strait Line drawn from the Point to the foot thereof.

The *Arches* of the *Signs* on the *Surface* of the *Dial*, are the Descriptions of the Parallels to the *Equinoctial*, which pass thro the 12 equal Divisions of the *Ecliptick Line*, which shew the beginning of the *Signs*.

CHAP.

CHAP. III.

To Mark the Points *of* Shadow.

Fig. 2. THe Practices which are taught in this Treatise, being founded on the *Points* of *Shadow* of the *Point* of the *Style* to be very small, it is very necessary to mark them very exactly; but it is very difficult because of the *Penumbra*: See here two ways by which it may be done.

The first is to fit a small round *Plate* to the *Point* of the *Style*, which may be parallel to the *Plane* of the *Dial*, whereof the *Center* may be joyned to the Point of the *Style*; then having drawn the Shadow of the said Plate on the Plane of the *Dial*, take the middle of that Shadow, which shall be the Shadow of the end of the *Style* at the same time when we observed the Shadow of the Plate.

The Second way is to make a small round hole in a little piece of Past-board or thin Plate, or other like body, and having applied it to the end of the *Style*, so as the *Center* of the Hole may be joined to the Point of the *Style*, and that the small Plate may regard the *Sun* perpendicularly; the light of the Sun shining thro the Hole

Hole ſhall mark a clear Circle or Oval D E in the Shadow of the Plate on the Plane of the *Dial*, which we draw on the ſaid Plane;and if it be an Oval,having drawn a ſtraitLine DPE from thePoint P, which is the foot of the *Style*, whereof *S* is the Point, which may paſs thro the *Center* of that Oval and cut it in D and E, or draw D G and E F parallel to one another, and making any *Angle* with D E, D G being made equal to D S, and E F equal to E S, the Line G F ſhall cut D E in the Point A, which ſhall be the Shadow of the Point of the *Style S*, at that time when the Oval was drawn: But we may take the *Center* of the Oval for the Point A, without falling into any ſenſible Error, as we may ſee by this Operation.

But if the Shadow be a Circle, the *Center* of that Circle ſhall be the Shadow of the Point of the *Style*.

Here Note, That we ought to make the Hole as ſmall as is poſſible, for ſo the Operation ſhall be more exact. It is ſufficient that we may ſee diſtinctly the *Figure* of the Light in the Shadow of the ſmall Plate.

CHAP. IV.

To Draw the Horizontal Line.

A *Style* the Point whereof is S, being planted upon the *Plane* of the *Dial*, we apply a Rule AS, ſo as one of the Edges thereof AS may be level, touching the Point of the *Style*, and that the end of that Edge of the Ruler may touch the *Plane* of the *Dial* at the Point A, which ſhall be one of the Points of the *Horizontal Line.*

By the Point A draw a level Line on the *Plane* of the *Dial*, which ſhall be the *Horizontal Line.*

There are ſo many ways to ſet a Ruler level, that I will preſcribe none in particular.

CHAP. V.

To find the Subſtylar Line, *two* Points *of* Shadow *being given in a certain condition.*

To find the Center *and draw the* Equinoctial Line; *the Declination of the* Sun, *and one Tract of* Shadow *being given.*

Fig. 4. LEt there be a *Style*, the Point whereof is S, and P the Foot, having marked the Point A, which let be the Shadow of the Point S, on the *Plane* of the *Dial*; on the Point P as a *Center*, and at the Diſtance P A, deſcribe the Circle A B, and when the Shadow of the Point S comes again to the *Circle* A B on the ſame day, at the Point B we mark that Point. Then draw the Line A B, and divide it into two equal parts in D, the ſtrait Line P D, ſhall be the *Subſtylar Line*, which ought to be at *Right Angles* to the Line A B. We may take divers Points, as A, for to find divers Points as B, and if the Operation be made exact, all the Points as D, ought to be in one and the ſame ſtrait Line with the Point P.

If the *Suns Declination* has changed conſiderably in the time which has paſſed between the two Obſervations of the Points of Shadow A and B; which may fall out when the *Sun* is near the *Equinoctial* Points, or when the *Style* P S is

B;

very high; or laſtly, when there has paſſed much time between the Obſervations, we ſhall not have the *Subſtylar Line* exactly.

The Demonſtration of the former Practice.

Suppoſing the Declination of the *Sun* not to be changed between the two Obſervations of the Points of Shadow A and B; by conſtruction the two *Triangles* A P S, B P S, are equal and alike; therefore the *Angle* P S A is equal to the *Angle* P S B; the *Sun* therefore was equally elevated above the *Plane* of the *Dial*, then when the two Points of Shadow were obſerved. Therefore the *Sun* was equally diſtant from the *Meridian* of that *Plane* at the times of the two Obſervations: Therefore the Line P D, which divides the Line or Arch A B into two equal parts, ſhall be the *Meridian* of the *Plane*.

To find the Center *of the* Dial, *and to draw the* Equinoctial Line, *knowing the place where the Shadow of the Point of the* Style *cuts the* Subſtylar Line.

If between the Points of Shadow A and B, we mark a Succeſſion of Points of Shadow, ſo that we may have the Point L where the Shadow of the Point of the *Style* meets with the *Subſtylar Line* between the Obſervations, having erected P F perpendicular to the *Subſtylar Line*, and equal to the height of the *Style* P S, let the ſtrait Line F L be drawn, and having made the *Angle* L F E equal to the Declination of the *Sun* at that

that time when the Point of Shadow L did meet with the *Subſtylar Line*, ſo as the Point E, or the meeting of the *ſtrait Line* F E with the *Subſtylar Line*, may be always towards the Convex part of the *Curvature* of the Line A L B, that Point E ſhall be the Point where the *Equinoctial Line* E G cuts the *Subſtylar Line*; which Lines ſhall interſect one another at *Right Angles.*

Then having drawn F C perpendicular to F E, the *ſtrait Line* F C determines the poſition of the *Axis* with the *Subſtylar Line*; and if it meets with it in the Point C, that Point ſhall be the Center of the *Dial*, and the Line F C ſhall be the *Angle* of *Inclination* of the *Axis* with the *Subſtylar Line*, which ſerves to place the *Axis*, and to find the other neceſſary Points for the Conſtruction of the *Dial.*

It is not neceſſary that the Point of Shadow L ſhould be taken on the ſame day when we obſerved the other Points A and B, it is ſufficient that we have the Declination of the *Sun* then when we make Obſervation of the Point L, and on which ſide the convexity of the *Tract* of the Shadow ſhall be on that day, to find the Point E.

Demonſtration.

The *Demonſtration* of this *Operation* is manifeſt, if we conſider that we have made the *Angle* L F E, and that the Line F C ought to be extended in the *Plane* of the *Meridian*, which is perpendicu-
lar

lar to the *Plane* of the *Dial*, and that the meeting of the Line D P C and the *Point* F, ought to be conjoyned in the *Point* of the *Style* S.

Another way of finding the Subſtylar Line *by the Amplitude of the* Suns *Riſing and Setting upon the* Plane *of the* Dial.

When the *Sun* begins to riſe on the *Plane* of the *Dial*, mark the Shadow of a Small Thread extended from the Foot of the *Style* to its *Point*, and do alſo the ſame when the *Sun* ſets on the *Plane* of the *Dial*, the *Angle* comprehended between theſe two Lines of Shadow, whoſe Vertex is at the Foot of the *Style*, being divided into two equal parts, ſhall give the *Subſtylar Line.*

This is manifeſt, for that *Angle* is the Sum of the Ortive and Occaſive Amplitudes of the ſame day, which we ſuppoſe to be equal.

The *Subſtylar Line* being placed, we may find the *Center* of the *Dial* by the Practice of the 11*th Chapter*, uſing only one Point of Shadow, and knowing the Declination of the *Sun* at the Hour where the Point of Shadow has been marked; and if the *Dial* have no *Center*, we may have the Inclination of the *Axis* with the *Subſtylar Line*; which ſhall ſerve inſtead of the *Center* for the placing of Hours: Alſo by the ſame Practice we may have the poſition of the *Equinoctial Line.*

CHAP.

CHAP. VI.

To place the Subſtylar *and* Equinoctial Lines, *and the* Center *of the* Dial, *and to determine the poſition of the* Axis.

Any Two Points of Shadow being given with the Declination of the Sun *at the time of Obſervation of the Points of Shadow.*

Fig.5. A *Style* being placed on the *Plane* of the Dial, whereof the Point may be S and P the Foot, and any two Points of Shadow A and B, taken at pleaſure.

Upon ſome certain *Plane* having made the *Angle d ſ a* equal to the Sum or Difference of a *Right Angle*, and that of the Declination of the *Sun* on that day on the which the Points of Shadow were mark'd, according as the Declination is North or South; for you would have a Point of the *Subſtylar Line* as Q which may anſwer to a Point of the *Axis*, which may be more North than the Point of the *Style*, you muſt make the *Angle d ſ a* equal to the Sum of a *Right Angle* and *Angle* of the Declination of the *Sun*, if the Declination be North, but equal to the Difference of a *Right Angle* and *Angle* of the Declination if it be South.

Having

Take *ſd* of any length, and make *ſa* and *ſb* equal to the *Intervals* S A, S B, from the *Point* of the *Style* to the *Points* of the *Shadow*; from the *Point* A as a *Center* at the diſtance *a d*, deſcribe Two Arches of *Circles* at T and L, and from *the Point* B, at the diſtance of the Line *b d*, *interſect* the former Arches at the *Points* L and T, then draw the *ſtrait* Lines L T and A B, which ſhall *interſect* one another at *Right Angles* in the *Point* O; and from the *Point* O as a *Center*, at the diſtance L O or O T, which are equal, deſcribe the *Semicircle* L D T.

Then draw the ſtrait Line P G K parallel to A B, interſecting L T in the *Point* G, and make G K equal to P S the height of the *Style*: and from the *Point* P, as a *Center*, at the diſtance of the Line *ſd*, deſcribe the Arch I, *cutting* the ſtrait Line L T in the *Point* I.

And from the *Point* K as a *Center*, and at the diſtance G I, deſcribe the Arch D R, *cutting* the *Semicircle* L T in the *Point* D: Then to the Line L T, let fall the perpendicular D Q, and draw the ſtrait Line Q P, which ſhall be the Subſtylar Line.

If the Point Q falls too near to the Point P, we may take *ſd* greater, and begin the work again to determine the Subſtylar Line more exactly.

If the *Declination* of the *Sun* be conſiderably changed between the *Obſervations* of the *Points* of *Shadow*, be it that the *Obſervations* be made on the ſame day or on different days, the *Angles*

zsa, *dsb* must be made according to the different *Declinations* at the times of *Observation*, and the *Angle dsa* being made as has been taught before for the time of the *Observation* of the *Point* of *Shadow* A, make *sa* equal to S A; also the *Angle dsb* being made for the *Observation* of the *Point* of *Shadow* B, make *sb* equal to S B.

Then from the *Points* P and Q of the *Substylar Line* raise the *Perpendiculars* P N equal to the height of the *Style* P S, and Q M equal to Q D; then if the *strait Line* MN being drawn meet with the *Substylar Line* in C, the *Point* C shall be the *Center* of the *Dial*, and the Line N E perpendicular to NM, meeting with the *Substylar Line*; the *Line* V E being drawn perpendicular to the *Substylar Line* passing thro the *Point* E, shall be the *Equinoctial Line* of the *Dial*.

We may observe that the *Point* D ought to be on the side with the Semicircle T L G, which is cut by the Line P G K, and which answers to the *Point* M of the *Axis*, which is supposed to be more toward the *North* than the *Point* S; the *Point* D may indifferently meet with the *Circle* on either side of the *strait Line* T L.

Also, We may observe that if the *strait Line* L T passes by the *Point* P, that Line shall be the *Substylar Line*, and it shall always pass by the *Point* P, if the *Points* of Shadow A and B are *Points* of the *Equinoctial Line*.

De-

Demonstration.

It is clear by this Construction that the Line ſd *represents the* Axis, *and* ſa *and* ſb *the Lines of the Shadow from the Point of the* Style, *which make with the* Axis ſd *an Angle equal to the Sum or Difference of a Right Angle and Angle of the Declination of the* Sun, *according to that which hath been prescribed in the Practice : Therefore if we conceive that the two Triangles* a ſd, b ſd, *each apart have their Points* a *and* b *in the Points of Shadow* A *and* B, *and their Points* ſ *joyned together with the Point* S *of the Style, if the Points* d *of each of these Triangles are also joyned together, these two Triangles in this Position compose a Pyramid* A B S d *whereof the Line* S d *is the* Axis *of the* Dial.

But to find on the Plane *of the* Dial *a Point which may answer perpendicularly to the Point* d, *as* P *answers to the Point* S, *we must consider in the Pyramide the Triangle* A B D *to move upon the Line AB, so as it is manifest that the Point* d *describes a Circle such as is* T L D, *on a* Plane *which is perpendicular to the* Plane *of the* Dial; *and the Intersection of those two* Planes *is the strait Line T L, which ought to cut the Line A B, which conjoyns the Points of Shadow A and B in the Point* O, *and* O T, O L *shall be equal : There remains nothing more but to mark the Point* d *or D upon the Circle L D T couch'd on the Plane of the* Dial.

The Plane *of the Circle* L D *T being perpendicular to the* Plane *of the* Dial, *if we conceive that the ſtrait* Line ſ d *moves upon its extreme*, ſ *being immovable, and put upon the Point of the* Style S, *and that the other extrem thereof* d, *may be always in its moving upon the* Plane *of the* Circle L D *T*, *it is evident that that extream* d *ſhall deſcribe a Circle R D upon that* Plane *which ſhall have for its Center the Point K, where the Line S K drawn from the Point of the Style S perpendicularly to the* Plane L D *T*, *meets with the ſame* Plane, *the which Line S K ſhall be equal to P* G: *but the Semidiameter of the Circle R D ſhall be equal to* G *I if P I be made equal to* ſ d; *but the interſection of the Circle* L D *T with the Circle R D, which is the Point D determines the poſition of the Point* d *of the* Axis. *So as D Q conſidered as perpendicular to the* Plane *of the* Dial, *coming from the Point* d *or D, does there mark the Point Q, which is one of the Points of the Subſtylar Line, ſeeing that the Point* d *is one of the Point of the* Axis.

It is alſo evident that the Plane N *M* C P *being ſuppoſed to be perpendicular to the* Plane *of the* Dial, *the Line* N *M does there repreſent the* Axis *in its poſition, the Point* C *the* Center *of the* Dial, *and conſequently the Line V E* ♈ *ſhall be the Equinoctial.*

Ano-

Another Practice upon the ſame Poſitions and Conſtructions.

The ſame Preparations being made as before, take two ſmall Rods of any firm Matter, as of Wood of a ſufficient thickneſs, or of Iron: and make them pointed at the ends, and equal in length to the ſtrait Lines *a d*, *b d*; it is not material whether they be ſtrait or crooked, if the Diſtances between their Points be equal to *a d* and *b d*.

Put one of the *Points* of that *Rod* which is equal to *a d* on the *Point* of *Shadow* A, and one of the Points of the other Rod to the Point of Shadow B, and joyn them together by their other *Points*; but ſo as the *Points* that are joyned together may approach or fall back from the *Point* of the *Style* without altering the other *Points* of the *Rods*, which are ſet on the *Points* of *Shadow* A and B; then we take with the Compaſſes or otherwiſe, the diſtance between the *Points* *a* and *d*, and ſet that diſtance between the *Point* of the *Style* and the *Points* of the *Rods* that are joyned together, ſo as that *Point*, as it may be more toward the *North* than the *Point* of the *Style*: by this means the *common Point* of the *Rods* being fixt ſhall be one of the *Points* of the *Axis* which ought to paſs by the *Point* of the *Style*, therefore the ſituation of the *Axis* ſhall be determined.

By this *commonPoint* of the *Rods* ſo fixt, which I call D, having drawn a Line perpendicular to the *Plane of the Dial*, which ſhall meet it in the *Point* Q, the Line P Q ſhall be the Subſtylar Line.

The *Point* C on the *Plane* of the *Dial* where it is met by the Line D S, drawn by the *Point* of the *Style* S, and by the end of the *Rod* D, ſhall be the *Center* of the *Dial*, and by the *Practice* of the 11*th Chapter* we may draw the Equinoctial Line: but if we have not the *Center*, we may draw it by the *Practice* of the ſame *Chapter*.

This Practice may ſerve to make you underſtand more eaſily the Demonſtration of the Conſtruction which is propoſed in that Chapter.

CHAP.

CHAP. VII.

To place the Subſtylar Line, *the* Center *of the* Dial *and the* Equinoctial Line.

One only Point *of* Shadow *being given, with the Declination of the* Sun *and height of the* Pole *above the* Horizon.

Fig. 6. HAving placed a *Style* upon the *Plane* of the *Dial*, whoſe *Point* may be S, and P the foot, and A one *Point* of *Shadow*, draw a *Horizontal Line* by the *Practice* of the 4*th Chapter*. And by the Point P draw the Lines B P H perpendicular to the *Horizontal Line* *h* H, and P Z parallel to H *h* and equal to the Height of the *Style* PS. Then from the Point H, where P H meets with the *Horizontal Line*, draw H Z and Z B perpendicular to Z H, which ſhall meet with H P at the Point B if the *Horizontal Line* paſſes not thro the Point P: Firſt let it meet at the Point B.

Upon ſome *Plane* make the *Angle* *d ſ a* equal to the Sum or Difference of a *Right Angle*, and

of the Declination of the *Sun* at the time when the Point of Shadow was obſerved, according to the Precepts which have been given in the *6th Chapter*, and make the *Angle d ſ b* equal to the Sum of a *Right Angle* and the height of the *Pole* above the *Horizon*.

Then taking the *Point d* at pleaſure on the Line *ſ d*, make *ſ b* equal to Z B and *ſ a* equal to the length of the Shadow from the *Point* of the *Style* S to the *Point of Shadow* A, and draw the *ſtrait Lines a d, b d*.

And by the *Points* A and B draw the *ſtrait Line* A B, aud from *the Point* B as a *Center*, at the diſtance *b d* deſcribe the Arch *ſ* L either above or below the Line A B; and likewiſe from the Point A as a *Center*, and at the diſtance *a d*, deſcribe the *Arch g d, cuteing the Arch* L *ſ* at the Point L, and from the Point L draw *the ſtrait Line* O L perpendicular to A B.

From the Point O as a *Center*, at the dſtance O L deſcribe *the Arch* D L.

And from the Point P draw *the ſtrait Line* P G K perpendicular to O L; and from the ſame Point P, at the diſtance *d ſ*, deſcribe *tho Arch* I either on the one or the other ſide of G, *cutting the Line* L O at the Point I.

Then make G K equal to P S the height of *the Style*, and from the Point K, at the diſtance G I deſcribe the Arch R D, cutting the Arch D L in D, and from the Point D draw *the ſtrait Line* D Q perpendicular to L O, and the Line P Q

which

which passes thro the Points P and Q shall be the *Substylar Line.*

If the Point Q be too near to P, we may find another by taking another Point *d* on the *Line* *s*, *d*, as has been said in the Practice of the Precedent *Chapter*; also we must have a regard to the other *Observations* which have been made upon the same Practice, by that which is there founded on the same Principles.

Consequently we place the *Equinoctial Line* and the *Center* of the *Dial* by the Practice of the 11*th Chapter*: but we have also here this advantage, that *the Line* which passes thro the Point B and thro the *Center* of the *Dial*, shall be the *Meridian Line.*

In the second place, if *the Horizontal Line* passes thro the *Point* P, or if the Point B be too far distant from the Point P, we must fasten another *Style* upon the *Plane* of the *Dial* whereof the Point may pass by the *Line* of the Plummet hanged from the Point S of the *Style*, the Point of that second *Style* being called B, we perform the *Operation* as before to find *the Lines* *d a*, *d b*: but we may use the small Rods, as has been taught in the fore-going *Chapter*, otherwise the *Operation* would be too long.

Demonstration.

This Operation is so like to the precedent; that the Demonstration does not much differ from it; for here instead of a Second Point of Shadow we have

have the Point B , and the Point b ſ of the Triangle bſd *being applied in B S, that Triangle moving upon B S and meeting the other Triangle* aſd *whereof the Points* aſ *are applied in A S, and which moves upon A S, ſo as the Points* dd *of the two Triangles may be joyned together; then the Axis which ought to paſs by the Points* Sd, *ſhall be ſtayed in that Poſition, as we have ſeen in the precedent Chapter: for in all the different Poſitions of the Triangle* bſd *moving upon B S, the Line* Sd *which repreſents the Axis, remains always elevated above the Horizen, with an Angle equal to the Elevation of the Pole above the Horizon, and it ſhall not be ſtayed in that Poſition, but by the meeting of the other Triangle* aſd, *ſo as the two Points* dd *of the two Triangles may be joyned together.*

The reſt of this Operation being altogether like to the precedent, we ſhall not here again repeat the Demonſtration.

CHAP.

CHAP. VIII.

To find the Center of the Dial, *and to situate the* Subſtylar *and* Equinoctial Lines.

One only Point *of* Shadow *being given, and the ſhorteſt* Shadow.

Fig. 7. HAving placed a *Style* on the *Plane* of the *Dial*, whoſe *Point* let be S and Foot P, and marked a *Point* of *Shadow* A, and divers other *Shadows* following one another, as GB, which may determine the length of the ſhorteſt *Shadow*, which is P B the *Semidiameter* of a *Circle*, that has for its *Center* the Foot of the *Style* P, which touches the Tract of the *Points* of *Shadow* GB in the Point B, and determines it exactly; which cannot be done by the meeting of the Two *Curves*, that is of the *Circle* NB and Tract of Shadow GB.

Therefore upon ſome *Plane* *pſ* equal to the height of the *Style* PS, draw *cpb* perpendicular to *ſp* and make *pb* equal to P B, and *ſb* ſhall be equal to SB; and prolong *ſb* to *a*, and let *ſa* be equal to the length of the Shadow SA.

Then

Then make the *Angle* $a\,s\,c$ equal to the Sum or Difference of a *Right Angle*, and *the Angle* of the *Declination* of the *Sun* at the time when the *Points* of Shadow were obſerved, following that which has been noted, in the Practice of the *6th Chapter*, the Line $s\,c$ meeting $b\,p$ in c draw the ſtrait Line $a\,c$.

Then on the *Dial* from the *Point* P. at the Diſtance $p\,c$ deſcribe the *Arch* C F, and from the *Point* of Shadow A, at the Diſtance $a\,c$ deſcribe the Arch D C, interſecting the Arch C F in the *Point* C, the *Point* C ſhall be the *Center* of the *Dial*, and the *ſtrait Line* C P ſhall be the Subſtylar which ſhall cut the Tract of Shadow in the *Point* B, where the *Circle* N B ought to touch it.

The *Subſtylar Line* being placed, we may draw the *Equinoctial Line* by the *Practice* of the *11th Chapter.*

Demonſtration.

This Practice is founded on the ſame Principles as the fore-going Operations, except that in this the Line ſ c *is the length of the Axis from the Point of the Style S to the* Plane *of the* Dial, *for the* Triangle c ſ b *moving upon the Line* ſ p *applied to* S P, *reſts in that Poſition as it ought to do to determine the Center* C *by the Triangle* a ſ c, *which moves upon* a ſ *applied to A S: but inſtead of theſe two Triangles it ſufficeth that the Lines* p c *and* a c *moving*

moving on the Plane *of the* Dial *about the Points* p *and* a, *may determine the* Center C, *as it is easie to conceive by the Construction.*

Note, That if in this Practice the Lines *b p* and *ſc* be parallel or make a very acute Angle, the *Axis* ſhall not meet with the *Plane* of the *Dial*, or elſe ſhall meet with it at a great diſtance from the Point P. Then you muſt uſe the following Practice.

Fig. 8. After you have made the ſame preparation as before, take *ſg* on the Line *ſc* of any length, and from the Point *g* draw a ſtrait Line *g m* parallel to *ſp*, which ſhall be alſo perpendicular to *b p*; from the *Point g* as a *Center*, at the diſtance *g a*, deſcribe the *Arch a e* interſecting *b p* in the *Point e.*

From the Point A as a *Center*, and at the diſtance *e m* deſcribe the Arch MQ, and from the Point P as a *Center*, at the diſtance *p m*, deſcribe the Arch M F interſecting the Arch QM in the Point M, and M P B ſhall be the *Subſtylar Line*,

Then draw the *ſtrait Line* P Z perpendicular to the *Subſtylar Line* M PB and equal to P S the heighth of the *Style*, and alſo M G perpendicular to the ſame *Subſtylar Line* M P B, and equal to *mg*, the Line G Z ſhall determine the Poſition of the *Axis* in reſpect of the *Subſtylar Line*, and Z E being drawn perpendicular to Z G meeting the *Subſtylar Line* in E the *Equinoctial Line*, ſhall

be

be V E perpendicular to the *Subſtylar Line* interſecting it in the Point E.

Demonſtratson.

There is no difficulty in the Demenſtration of this Practice after that which has been explained before, we muſt only conceive here the Triangle e m g *to be perpendicular to the Plane of the* Dial, *that it is Right angled at the Point* m, *and that it is moved about the* Point e *which is put upon the Point* A, *ſo as the Line* e m *is always on the Plane of the* Dial, *and alſo that the Trapezium* p ſ g m *is movable about the Line* p ſ *which ought to be applied to the Style P S, ſo as the Triangle* e m g *and the Trapezium* p ſ g m *may meet in their common Line* m g *or* M G, *the Line* M G *ſhall be another Style anſwering to the firſt, the Point* M *ſhall be the Foot thereof, and* G *ſhall be the Point, and the Line that paſſes by the* Points *of theſe two Styles* S *and* G, *ſhall be the Axis, and conſequently the Line* M P *that paſſes by the Feet of theſe correſpondent Styles ſhall be the Subſtylar Line.*

It is alſo manifeſt that the Line Z G *determines the Inclination of the Axis in reſpect of the Subſtylar Line.*

We may take divers Points of Shadow as A, to confirm this Operation, as we have done in the Practice of the *5th Chapter*, the ſhorteſt Shadow P B remaining always the ſame.

It

It is to be obſerved, That if the *Suns* Declina-
tion hath conſiderably changed between the Obſervation of the Point of Shadow A, and the ſhorteſt Shadow P B, the Angle *pſc* muſt be made as we have taught before for the time of he ſhorteſt Shadow, and then draw *ſa*, which nakes with *cſ* the required Angle for the time f Obſervation of the Point of Shadow A: for t is not neceſſary that the Obſervations of the oint of Shadow A, and the Point of the ſhorteſt hadow B, ſhould be made on the ſame day.

CHAP. IX.

o find the Center *of the* Dial, *and to draw the* Subſtylar *and* Equinoctial Lines.

wo Points *of* Shadow *being given, with the Declination of the* Sun *at the times when the* Points *of* Shadow *were obſerved.*

ig. 9. A *Style* being placed on the *Plane* of the *Dial* whereof the Point may be and the Foot P , and two Points of Shadow and B being obſerved.

Make

Make the *Angle e ſ a* equal to the Sum or Difference of a *Right Angle* and the Declination of the *Sun* at the hour when we obſerved the Point of Shadow A, following the Rules that we have given in the Practice of the *6th Chapter.*

Let ſeveral *ſtrait Lines* be drawn, as A E, toward the place where we ſuppoſe ſomething near they ought to meet the *Center* of the *Dial*, and from the *Center* A at any diſtance deſcribe the Arch of a *Circle* D R, interſecting A E in the Point D. And having made *ſ a* equal to S A, from the *Center* Q at the diſtance *a r*, equal to A R deſcribe the Arch *d r.*

Then from the Point *ſ* as a *Center*, at the diſtance S D, deſcribe the Arch *d n*, cutting the Arch *r d* in *d*, and draw the *ſtrait Line a d e* to meet with the *Line ſ e* in the Point *e*, and make A E equal to *a e*, and having found ſeveral Points as E, draw the *Curve Line* E C by all thoſe Points. If we perform the ſame Operation by the Point of Shadow B, and we ſhall find another *Curve Line* F C, which interſecting the *firſt Curve Line* in the Point C, determines the Center of the *Dial.*

And C P ſhall be the Subſtylar Line.

We may draw the Equinoctial by the Practice of the 11*th Chapter.*

The Demonſtration of this Practice is ſo eaſie, that it needs no Explication.

For we may clearly ſee that the Interſection E of the two *Curve Lines* is the Interſection of the ſides

ſides of two Angles *a ſe* and *b ſe*, which are conjoyned by their common Line *ſe*, and therefore the ſides *a ſ* and *b ſ* on which theſe Angles move, are applied to *A S* and *B S*, and the common ſide of theſe Angles *ſe* which is the *Axis*, being prolonged, meets with the *Plane* of the *Dial* in C, which ſhall be the *Center* thereof.

CHAP. X.

To find the Center *of the* Dial, *and to draw the* Subſtylar *and* Equinoctial Lines.

Any Two Points *of* Shadow *being given, with the Declination of the* Sun *at the time of Obſervation of the* Points *of* Shadow.

Fig. 10. LEt P S be a Style, whereof P may be the Foot and S the Point, and A and B two Points of Shadow. From the Point A, at the Diſtance A S, deſcribe the Arch Q Z: Alſo from the Point B, at the Diſtance B S, deſcribe the Arch R Z, interſecting the Arch Q Z in the Point Z, and draw the ſtrait

Lines, Z B and Z A, and the Line Z P forth at length, then make ZH equal to Z B, and draw the ſtrait Line B H, which being divided into two equal parts in E, let Z E be drawn to meet with A B in the Point F.

Then erect P N perpendicular to Z L, and equal to the height of the *Style* P S, and draw G O and O L at *Right Angles*, and O L ſhall meet with Z P L at the *Point* L; the Line L F ſhall be one Line which ought to paſs by the *Center* of the *Dial*.

If the *Center* of the *Dial* that is to be found, be more *North* than the *Point* of the *Style*; and if the *Declination* be more *North*, having made the *Angle c ſ b* equal to the Sum of a *Right Angle*, and of the *Declination* of the *Sun* at the time when we have marked the *Points* of *Shadow*; but if the *Declination* be *South*, having made *the Angle c ſ b* equal to the difference of thoſe Two *Angles*.

And if *the Center* of the *Dial* required be more *South* than the Point of the *Style*, and the *Declination* be *North*, having made the *Angle c ſ b* equal to the Difference of a *Right Angle* and *Declination* of the *Sun*, but if the *Declination* be *South*, having made that *Angle* equal to the Sum of the two *Angles*.

Let *ſ b* be made equal to S B, and from the *Point b* having drawn *b d* perpendicular to *c ſ* prolonged, if it be neceſſary: from the *Point* B, and at the Diſtance *b d* deſcribe the Arch D *cutting* Z F in D, then making *d e* equal to D E, draw the *ſtrait Line ſ e*.

From

From the *Point* P draw P I N perpendicular to F L, and make I N equal to I S, and draw F N, then draw N C, making an Angle with F N equal to the Angle *e ſc*, the Line N C ſhall meet with the Line F L, at the *Point* C, the *Center* of the *Dial* if it have a *Center*, and the ſame *Line* C N determines the inclination of the Axis upon the *Line* C F on a *Plane* which is inclined to the *Plane* of the *Dial* towards P, with an Angle equal to SIP.

And C P ſhall be the *Subſtylar Line*, the *Equinoctial Line* may be found by the *Practice* of the 11*th Chapter*.

But if the *Dial* have no *Center*, which is known by the *Line* N C being parallel to the *Line* *F L*; or if the *Center* be at a great Diſtance from the *Point* P: Then draw the *Line* T V X parallel to P N, and make V X equal to V T, and draw N X *cutting* V I in M, and draw P M meeting T V in Y, and make V *y* equal to V Y, and *y* P ſhall be the *Subſtylar Line*. The *Equinoctial Line* is drawn by the Practice of the 11*th Chapter*.

To find the Center by three Points of Shadow.

Having marked three Points of Shadow, we may find the *Line* *F L* by two of thoſe Points which ſhall paſs thro the *Center*, and by one of thoſe Points and the third, we may find alſo another *Line* which ſhall paſs thro the *Center*; and the

Point where these two *Lines* do meet, shall be the Center of the *Dial*.

This Practice may be performed without knowing the Declination of the *Sun*.

The Center being found, and consequently the Substylar *Line* which passes thro the Center and by the Point P, the Equinoctial Line is drawn by the Practice of the 11*th Chapter*.

Demonstration.

Fig. 11. To demonstrate this Practice, we must suppose that the Triangle A S B in the 11th *Figure* to be the same with the Triangle A Z B of the 10*th Figure*, so as the Point Z is joyned to the Point of the Style, and that the *Line* A B remains in its position.

It is evident that all the *Planes* that are perpendicular to the Plane of the Triangle S A B and which passes by the Point of the Style S, intersect one another in the Line S L, which is raised perpendicular to the Point S upon the Plane of that Triangle: Therefore the Plane *L S F* is perpendicular to the Plane S A B, and I say that the Axis of the *Dial*, and consequently the Center, shall be in that Plane *L S F*.

For the *Line* S F divides the Angle A S B into two equal parts; and S B being equal to S H, the two Triangles S B D, S H D, being equal each and like to the Triangle *s b d* of the 10*th Figure*, and being joyned together by their common side S D, that Line S D shall be the Axis in its true position, and the Arch of a Circle B H de-

described from the Center D at the Distance D B, shall be the parallel of the Sun, for the Angle S B D is equal to the Suns Declination.

But in these two *Equicrural Triangles* H S B, HDB, because that the common Base B H is equally divided in the Point E, the Plane that passes by the Lines E S, E D, shall be perpendicular to the Planes of the two Triangles, therefore the *Plane* which is perpendicular to the *Planes* of the two *Triangles*, and that passes by their Vertical Points S, D, shall carry the Axis: but the Plane *that* passes by the *Point* S, and by the *Point* E, and which shall be perpendicular to the Plane of the Triangle HSB, shall be also perpendicular to the Plane of the other Triangle.

Therefore the *Axis* shall be in the *Plane* F L S that is perpendicular to the *Plane* of the *Triangle* A S B, and which passes by the strait Line S E F, and consequently the *Axis* meets with the strait Line F L in some Point, or is parallel to it, and the Point C where the *Axis* meets with the Line F L shall be the *Center* of the *Dial*, which was to be demonstrated.

Then to demonstrate the Practice which serves to find the *Center* of the *Dial*, we must consider that B H being the Chord of the Arch of the parallel of the *Suns* Declination contained between the two Lines of Shadow S A, S B, and the Line B D being equal to the Line *b d*, and *d e* being equal to D E, the *Triangle* *s d e* is the same with the *Triangle* S D E of the 11th Figure.

There-

Therefore the *Angle* cse is that which the *Axis* ought to make with the Line F L on a *Plane* which passes by that Line F L, and by *S* the Point of the Style.

Therefore in the 10th Figure the *Triangle* F C N having the Line I N equal to I S of the *Triangle* F S C, and being both of them perpendicular to the common Line I L, and by the same Point I, and the Angle F N C being equal to the Angle esc, the Point c shall be the *Center* of the *Dial*, if the Line N C meet with the Line I L, which was that which was at last to be demonstrated. We suppose that in this Practice the Difference of Declination between the Observations is not considerably changed.

As concerning the Practice which serves to find the *Substylar Line*, when the Point C is at a great Distance from the Point P, or when the *Dial* has no *Center*; it is evident by Construction that T V being to V Y as N I is to I P, the Lines N T and P y ought to meet one another in the same Point C, which shall be the Center; or they shall be parallel to one another, and then the *Dial* shall have no *Center*.

CHAP. XI.

o find the Center *of the* Dial, *and to draw the* Equinoctial Line.
he Substylar Line *being drawn, and one* Point *of* Shadow *being given, with the* Declination *of the* Sun.

AND

o draw the Equinoctial Line *the* Center *of the* Dial *being found.*

AND

o find the Center *of the* Dial, *the* Equinoctial Line *being drawn.*

ig. 12. LEt a *Style* be placed on the *Plane* of the *Dial*, whose Foot let be P, and oint S, and Point of Shadow A, and Substyr Line C P.

Make the Angle *d s a* equal to the Sum or ifference of a *Right Angle*, and of the *Suns* eclination, as we have done in the Practice of e 6*th* Chapter, *s a* being made equal to S A, ke any Point as *d* upon the Line *s d*, and draw e strait Line *a d*.

 From

From the Point A draw the ſtrait Line AR perpendicular to the Subſtylar Line C P, and from the ſame Point A as a Center, and at the Diſtance *a d*, deſcribe the Arch N cutting the Subſtylar Line in N.

From the Point R as a Center, and at the Diſtance R N, deſcribe the Arch N D; erect the perpendicular P Z at *Right Angles* to the Subſtylar Line, and equal to P S the height of the Style, then from the Point Z as a Center at the Diſtance *ſd*, deſcribe the Arch G D cutting the Arch N D in D: The Line Z D determines the ſituation of the *Axis* in reſpect of the Subſtylar Line, and if it meets with the Subſtylar Line as at the Point C, that Point C ſhall be the Center of the *Dial.*

Then from the Point Z draw the ſtrait Line Z E perpendicular to Z D, meeting the Subſtylar Line in E, the Line V E perpendicular to the Subſtylar Line drawn through the Point E, ſhall be the Equinoctial Line.

If the Center of the Dial were given, the Line C P ſhall be the Subſtylar Line, and P Z being perpendicular to the Subſtylar Line, and equal to the height of the Style, having drawn C Z and Z E perpendicular to C Z, meeting the Subſtylar Line in E, the Line V E being drawn perpendicular to the Subſtylar thro the Point E, ſhall be the Equinoctial Line.

If the Equinoctial Line were given, draw the ſtrait Line P E by the Foot of the Style perpen-

pendicular to the Equinoctial Line VE, and PE ſhall be the Subſtylar Line; and having made PZ perpendicular to the Subſtylar Line PE drawn by P the Foot of the Style, and equal to the height of the Style, having drawn ZE and ZD perpendicular to ZE, the Line ZD determines the inclination or poſition of the *Axis* with the Subſtylar Line; and if ZD meets the Subſtylar Line as at C, that Point C ſhall be the Center of the *Dial.*

The Demonſtration of theſe Three Practices has nothing in it that deſerves to be explain'd after the fore-going Demonſtrations; for we may ſufficiently know that the Triangle *E Z C is the ſame with* E S C *which was upon the* Plane, *which was drawn by the Point of the* Style, *and by the* Subſtylar Line.

CHAP.

CHAP. XII.

To draw the Equinoctial Line, *and the* Subſtylar Line, *and to find the* Center *of the* Dial.

Any Two Points *of* Shadow *being given, with the* Declination *of the* Sun.

Fig. 13. LEt a *Style* be ſet on the *Plane* of the *Dial*, whoſe Foot may be P, and Point S, and let the Two Points of Shadow be A and B.

Make the *Angle aſf* equal to the Declination of the Sun at the time when we have marked the Point of Shadow A, and make *ſa* equal to S A, and on the Point A as a Center deſcribe the Arch M L at any Diſtance; and on the Point *a* deſcribe the Arch *lm* equal to the Arch L M.

Then by the Point A draw ſeveral ſtrait Lines as A L, A M, toward the place where the Equinoctial ought to paſs, which may be known by the Declination being North or South, and knowing near the poſition of the Wall in regard of the *Axis*.

Then

Then from the Point *ſ* as a Center, and to the Diſtance S M, deſcribe the Arch *m* cutting the Circle *l m* in *m*; then having drawn *a m* meeting *ſ ſ* in *ſ*, make A F equal to *a ſ*: Alſo make *ſ l* equal to S L, having drawn *a l* cutting *ſ ſ* in *d*, make A D equal to *a d*, and ſo of all the other Lines which we have drawn by the Point A, by which we find the Points as F D, and by all thoſe Points as F D we draw a Curve Line F D.

We do the ſame thing for the Point of Shadow B, about which we draw a Curve Line I G, and the ſtrait Line D C, which touches the two Curves, ſhall be the *Equinoctial*.

By the Practice of the afore-going Chapter we draw the Subſtyle, and find the Center of the *Dial* (if it has a Center) with the inclination of the *Axis* to the Subſtylar Line.

Demonſtration.

The Demonſtration of this Practice is not difficult to thoſe who underſtand the Conique Sections; they will preſently conceive that theſe Curve Lines, which are traced about by the Points of Shadow, are the Sections of an upright Cone, whereof the Angle at the Vertex of the Triangle *drawn by the* Axis *is double the Declination of the* Sun.

The

The Axis of the Cone is a strait Line drawn from the Point of the Style to the Point of Shadow.

Then if we conceive that there is a Plane *which touches these two Cones both together, and which passes by their Vertex, as is the Point of the Style; that* Plane *shall be the Equinoctial; for the* Plane *that passes by the Axis of the Cone, and by the Line, where the touching* Plane *meets the Cone, shall be perpendicular to the touching* Plane, *as may easily be seen; and the Angle contained between the Axis of the Cone and the Tangent Line, being the Declination of the Sun by Construction, the touching* Plane *shall necessarily be the* Plane *of the Equinoctial.*

CHAP.

CHAP. XIII.

To find the Points *of the* Hours *of* 6 *and* 12 *on the* Equinoctial Line, *and to draw the* Meridian Line.
The Equinoctial *and* Horizontal Line *being given.*
Or to draw the Meridian, *the* Center *of the* Dial *being given.*

Fig. 14. LEt P S be the height of the Style, whereof P is the Foot and S the Point, and let N L be the Horizontal Line, and M N the Equinoctial Line; the Point N where the Equinoctial Line meets with the Horizontal Line, is the Point where the Line of the Hour of 6 interſects the Equinoctial Line.

From the Center N, and at the Diſtance N S, equal to the height of the Style, deſcribe the Arch K H, and taking any Point as O in the Equinoctial Line for a Center, at the Diſtance O S deſcribe the Arch I H interſecting the Arch K H in H, then draw the ſtrait Line N H, and H M perpendicular to N H; the Point M where H M meets the Equinoctial Line, is the Point where the Meridian Line ought to interſect the Equinoctial Line.

Then

Then having hanged up a Line with a Plummet F, ſo as the Line may paſs by S the Point of the Style, mark any Point as C, on the *Plane* of the *Dial*, ſo as you may ſee with one Eye the Points M and C both hid together by the Line of the Plummet, and this is that which we call bourning, and the Line M C ſhall be the Meridian Line.

But if the *Center* of the *Dial* were given, and that it were the *Point* C, we muſt mark ſome *Point* as M on the *Plane* of the *Dial*, which we may ſee to paſs by the *Line* of the *Plummet* with the *Point* C, and the *Line* C *M* ſhall be the Meridian.

We may alſo draw this Meridian Line in the Night with a Candle, in holding it at a diſtance from the Line of the Plummet, ſo as the Shadow thereof may paſs by M or by the Point C which of them is given: For the Shadow of that Line ſhall be the Meridian Line.

Demonſtration.

The Horizontal Circle cuts the Equinoctial Circle into two equal parts and equally diſtant from the meeting of the Meridian with thoſe Circles; for the Equinoctial is perpendicular to the Axis, and the Horizontal is perpendicular to the Vertical Line, and the Meridian paſſes by the Axis and by the Vertical Line; Therefore the Horizontal Line cuts the Equinoctial Line in the ſame Point where the Hour Circle of 6 *meets with them, which is* 90 *degrees from the Meridian on the ſame E-*

quinoctial

quinoctial Circle; therefore the Line and Plummet being a part of the Vertical, the Plane *that passes by that Line, and by a Point of the Meridian or of the Axis, shall be the* Plane *of the Meridian, the Intersection whereof with the* Plane *of the* Dial *is the Meridian Line.*

CHAP. XIV.

To draw the Meridian Line, *and to find the* Point *of the* Hour Line *of six on the* Horizontal Line.
Only one Point *of* Shadow *being given, with the height of the* Pole, *and the Declination of the* Sun.

Fig. 15. A *Style* being given, whereof let S be the Point, P the Foot, being placed on the *Plane* of the *Dial*, draw the Horizontal Line H *h*, and from the Point P draw P H perpendicular to H *h*, and draw P Z parallel to H *h*, and equal to P S the height of the *Style*, and make H E equal to H Z.

Then having mark'd the Point of Shadow A, as far from Noon as it is possible, hang a Plummet T, so as the Line thereof may pass by S the Point of the *Style*, and bourn it (as we did in the fore-going Practice for the Meridian) mark the

the Point *b* on the Horizontal Line, by which you ſee the Line paſs, then when it alſo paſſes by the Point of Shadow A, and then draw the ſtrait Line *b* E.

Then draw an Arch of a Circle *z o f* on the Center *c* at any Diſtance, and make *z o* equal to the height of the Pole above the Horizon, and draw the Lines *o c*, *z c*, to *c* the Center of the Circle.

Let the Arch *o m* be made equal to the Declination of the *Sun* at the time when the Point of Shadow was made toward Z, if the *Sun* be in North Signs, and toward *f* if it be in South Signs, for thoſe that have their *Zenith* in the North part of the Sphere; but on the contrary for thoſe that have it in the South part; and draw *a m* parallel to *o c*.

Draw *c f* perpendicular to *c z* from the Center *c*, and make the *Angle d c f* equal to the *Angle b* S A, and draw the Line *d e* parallel to *f c*, meeting *c z* in *e* and *a m* in *a*.

On the *Point* E as a *Center*, at the Diſtance *d e*, deſcribe the *Circle* B D, meeting *E b* (prolonged if it be neceſſary) at the *Point B* make B M equal to *d a*, and from M raiſe D M perpendicular to E B, interſecting the Circle BD in D, then draw E D (prolonged if it be neceſſary) and the Point F where it interſects the Horizontal Line, ſhall be *the Point* of the Meridian upon the Horizon.

And

And E G being drawn perpendicular to E D, gives the Point G where the Hour Line of six meets with the *Horizontal Line.*

It must be observed that the Line M D which is drawn perpendicular to B E, may meet with the Circle B D on either side of the *Point* B; but you must take care that if the *Point* of Shadow A, is mark'd before Noon, to make use of the Point D, which is on the right hand of the Point B as in the *Example*, and if the *Point* A were marked after Noon, you must take the *Point* D where M D meets the Circle on the left hand of B to have the position of the Meridian Line; if D E meets not the *Horizontal Line*, but then when it is prolonged toward E, the Point F shall appertain to the Line of Midnight: All this ought to be understood concerning those that have their *Zenith* on the North side of the *Equinoctial*, for it is contrary with those which have their *Zenith* in the *Southern Hemisphere.*

If the Line E D meets not the *Horizontal Line* being likewise prolonged towards E, then the *Dial* shall have no Line of Mid-day nor of Mid-night, and the *Plane* of the *Dial* shall be either Oriental or Occidental.

It is also to be observed that the *Angle* H E F made by the *Line* E D with the *Horizontal Line* E H, is the *Angle* of the *Declination* of the *Plane.*

By the fore-going *Practice* the *Meridian Line* or the Line of Mid-night may be drawn by the *Point* F.

Demonstration.

The Figure f m z c *is a Projection of the Sphere on a* Plane *by Lines parallel to one another, which Projection is called the* Analemma; *the Circle* f m z *represents the Meridian, and the Point* Z *the Zenith,* o c *the Equinoctial Line,* a m *the parallel of the Suns Declination at the time when the Point of Shadow A was mark'd,* c f *the Horizontal Line,* d a e *the parallel of the Suns Altitude at the same time, and the Point* a *determines in the Sphere the place where then the* Sun *was.*

If a Verticle Circle z a g *be conceived to pass through the Point* a *to the Horizon in* g, *it is manifest that the Line* f g *in the Circle whose Radius is* f c, *shall be the Versed Sine of the Angle which the Verticle Circle* z a g *makes with the Meridian Circle* a d f, *and that* d a *shall be the same Versed Sine in the Circle, whose Radius is* d e.

Therefore if we conceived the Plane h, *E F D to be the* Plane *of the Horizontal Circle, we may see by Construction that the Line E F D is the Meridian on that* Plane, *and that the Point F is in the* Plane *of the* Dial, *the Point of the Meridian where it meets with the Horizon, and that the Point G is the Point where the Hour of six cuts the Horizon, seeing that F E G is a Right Angle; for B M equal to* d a *is the Versed Sine of the Angle B E D, which ought by consequence to be equal to the Angle which is made by the Meridian with the Verticle Circle.*

CHAP. XV.

To draw the Meridian Line.
Two Points *of* Shadow *being given in a certain condition.*

Fig. 16. HAving fixt a *Style* on the *Plane* of the *Dial*, whereof let P be the Foot and S the Point, draw the *Horizontal Line* H *b* by the Fourth *Chapter*, and from the Point P draw P Z parallel to *h* H, and equal to P S the height of the *Style*, and having drawn the *strait Line* Z H ; draw Z C perpendicular to it.

Then the *Point of Shadow* A being mark'd 3 or 4 hours before Noon, to the intent that the Operation may be more exact, let A B be drawn perpendicular to Z C.

Then make B E equal to B C ; and C D equal to E A, and draw Z D.

By the place where you judge something near that the *Shadow* of the *Point* S will arrive when the *Sun* shall be so far from Noon as it was when the *Point* of *Shadow* A was mark'd, draw many *strait Lines* as Q F perpendicular to H P, and by the *Points* F where they intersect H P, draw I F R perpendicular to Z C, and making F G equal to F R,

from the Point G as a Center, and at the Distance R I describe an Arch Q, which shall cut F Q in the Point Q, and so we may find as many Points Q as we will; and by all these Points drawing a Curve Line Q Q, we mark the second Point of Shadow O, then when the Shadow of the Point of the *Style* S meets with the *Curve* Q Q.

Then draw the Line A O, and on the Point A as a Center, and at the distance A S describe the Arch X Y, also from the Point O at the distance O S describe the Arch X N, cutting the Arch X Y in X, and draw X A, X O; then divide the Angle A X O into two equal parts by the Line X M, which shall meet with A O in M, the Point M shall be one of the Points of the Meridian Line, and by the Practice of the 13*th Chapter* we may draw the Meridian Line, which ought to pass by the Point M.

We suppose in this Practice that the Suns Declination has not changed considerably between the two Observations of the Points of Shadow A and O.

Demonstration.

The Curve Line O Q Q which ought to pass by the Point V, as is easie to understand by the Description, shall also pass by the Point A, if we make the Description thereof on the other side of the Line F H; and it is the Section of a Right Cone,

Cone, whereof the Vertex is the Point S, and D Z C the half of the Triangle which passes by the Axis, couch'd upon the Plane *of the* Dial, *and whereof Z C represents the Axis, the Angle D Z C is the Complement of the height of the* Sun *above the* Horizon *at the time when the Point A was mark'd, and the Lines as I R are the Sections of the* Planes *parallel to the Base of the Cone with its Triangle by the Axis, and upon which the Section of the Cone is a Circle, and the Intersections of those Circles with the* Plane *of the* Dial *are the Points Q, which are in the Superficies of the Cone and consequently in the Conique Section A V Q Q whereof H V P is the Axis: But this Conique Section represents the parallel to the Horizon where the Sun was found at the time of the two Observations of the Shadow in A and O; Therefore the Line X M which divides the Angle A X O into two equal parts, is the Intersection of the Meridian with the Plane A O S, for that Line shall divide the Chord of that* Angle, *the Point M is therefore one of the Points of the Meridian Line on the* Plane *of the* Dial, *in supposing the Declination of the* Sun *to be the same for the two Points of Shadow A and O.*

CHAP. XVI.

To find the Center *of the* Dial *or determine the Inclination the* Axis *with the* Meridian, *to draw the* Substylar *and* Equinoctial Lines.
The Meridian *being found, and the height of the Pole being given.*

Fig. 17. LEt the Point P be the Foot of the Style whereof S is the Point, and let M T be the Meridian. Draw the Horizontal Line H *h* in tersecting the Meridian Line in T; and from the Point T as a Center at the distance T S describe the Arch B A, and feom any Point of the Meridian as M, at the Distance M S describe the Arch D A intersecting the Arch B A in the Point A, and draw the strait Line A T, then make the Angle T A C equal to the height of the Pole above the Horizon which is here given; if the Line A C meets with the Meridian, as at the Point C, then C shall be the Center of the Dial, and the same Line A C in this Position determines the Inclination of the Axis with the Meridian Line; And *c* P shall be the Substylar Line.

From

From the Point A draw AE perpendicular to AC, interſecting the Meridian Line in E, and the Line EV drawn thro the Point E perpendicular to the Subſtylar Line CP, ſhall be the Equinoctial Line.

But if the *Dial* have no Center, draw the Line RPG at pleaſure thro the Point P meeting with the Meridian in G, and with the Line AC in R, then draw another Line ON parallel to RG, meeting the ſame Lines at the Points N and O; and draw the ſtrait Lines RN and GO interſecting one another in the Point F, and make R*p* equal to GP, and draw *p*F to meet with the Line ON in Q; and PQ ſhall be the Subſtylar Line, and the Equinoctial Line is found as before.

Demonſtration.

By the Conſtruction it is manifeſt that the Line AT *repreſents the Interſection of the Horizontal* Plane *on the* Plane *of the Meridian, therefore the* Angle TAC *being equal to the height of the Pole above the Horizon, it is manifeſt that the Line* AC *determines the Inclination of the* Axis *with the Meridian MT; therefore conſequently the Point* C *ſhall be the Center of the* Dial *if the Line* AC *meets with the Meridian Line: But the Line* AE *which is perpendicular to* AC *which*

represents the Axis, *shall be the Intersection of the* Plane *of the Equator and Meridian; therefore the Point E shall be one of the Points of the Equinoctial Line on the Meridian Line; but the Equinoctial Line ought always to be perpendicular to the Substylar Line, wherefore E V shall be the Equinoctial Line.*

If we have not the Center of the Dial, *we have drawn R P G and O Q N parallel to one another, and R N, G O, intersecting in F, and* p *F Q passing thro the same Point F, then by reason of the like* Triangles *we have R G to O N as R* p *or G P to Q N; therefore P Q, M T, R O ought to meet in one and the same Point C, which shall be the Center of the* Dial, *or else they shall be all three parallel to one another, and then the* Dial *shall have no Center.*

CHAP.

CHAP. XVII.

Remarques and Practices for divers Abridgments in the Operations of the fore-going Chapters.

I.

Fig. 18. HAving found the Subſtylar Line *p e* and the Equinoctial Line *e u* for the Style *ſp*, if you would remove the Subſtylar and Equinoctial Lines to another place of the *Plane* of the *Dial*; the Line P E parallel to *p e* ſhall be another Subſtylar Line, and V E parallel to *e u* or perpendicular to P E paſſing thro any Point of the Subſtylar Line P E, ſhall be the Equinoctial Line, and we determine by the following Method the Poſition of a Style for the two Lines P E, E V, whereof the height ſhall be given of any length, or we will determine the height of a Style whereof the Poſition ſhall be given upon the Subſtylar Line P E.

Firſt, Let the Line A R be given for the height of the Style, which ought to be ſet for the Subſtylar and Equinoctial Lines P E, E V.

Make

Make E P equal to *e p*, and ſet it the ſame way (that is the Point P muſt be ſet above the Point E if the Point *p* be above the Point *e*, and below it, if it be below it) and make E Z equal to *p ſ*, and E Z equal to R A given, and draw z P and Z R parallel to Z P, meeting E P in R; and the Point R ſhall be the Foot of the Style, the height whereof R A is given.

Therefore if you fix a Style, whereof the Foot may be R, and the Diſtance between the Point thereof A and Foot R, may be the height equal to the Line Z E, the Propoſition is ſatisfie d.

But if the Point R were given for the Foot of the *Style*, and the height were required: draw P Z as before, and by the Point R draw R Z parallel to P z, and E Z ſhall be the height of the Style whoſe Foot is the given Point R.

II.

The Subſtylar Line C E, the Equinoctial Line E V, and the Meridian Line C M, anſwerable to the Style S P being given, we may take what Point we will in the Subſtylar Line as K, to the Center of the Dial, without altering the Subſtylar Line or Equinoctial, and the Line K *m* drawn parallel to C M ſhall be the Meridian Line, but the height and poſition of the Style muſt be changed by the following Method.

Make

Make E z equal to P S, and draw M P and P Z, and draw *m* R and R Z parallels to M P and P Z, and the Point R ſhall be on the Subſtylar Line, which is the Foot of the Style, whereof the height is R Z perpendicular to the Point R on the Plane of the *Dial.*

III.

If the Subſtylar Line C E were given with the Meridian Line C M anſwering to the Style P S, we may take any Point as *p* to be the Foot of a Style, whereof the height is to be determined; or the *Style being* given of any height to determine the Poſition of the Foot *p* without changing either the Meridian or the *Center* of the *Dial.*

If the Foot of the *Style* *p* be given, and we are to determine its height, by the Foot of the *Style* P, put for the finding of the Meridian and Subſtylar Line, let there be drawn P S perpendicular to the Subſtylar Line and equal to the height of the ſame *Style*, and let C S be drawn by the *Center* of the *Dial*; and from the given Point *p* let *p ſ* be drawn parallel to P S, till it meet C S in the Point ſ, and *p ſ* ſhall be the length of the height of the *Style*, which ought to be placed at the Point *p*, and the Meridian C M and the Center of the *Dial* C are not changed.

But

But if *p ſ* were given for the height of the *Style*, it muſt be put upon P S prolonged if it be neceſſary in P *z*, and *z ſ* muſt be drawn parallel to C P to meet with the Line C S in *m ſ*, and *ſ p* being drawn parallel to S P, ſhall give the Point *p* on the Subſtylar Line for the Foot of the Style required, whereof the height is given.

IV.

Fig. 19. The Meridian C M being given, with the Equinoctial Line E M, we may find another Equinoctial as *e m*, without changing the *Meridian*, the which Equinoctial *e m* ſhall make the Angle *e m* C with the Meridian C M equal to the Angle E M C, but we muſt find another *Style* by the following Method.

If the *Center* of the *Dial* be given at the Point C, by the Foot of the *Style* P which have ſerved to find the Meridian and the *Center* C, having drawn P S perpendicular to the Subſtylar Line C P, and equal in length to the ſame *Style*, let *e ſ* be drawn parallel to E S, and from the Point *ſ* draw *ſ p* parallel to S P, meeting the Subſtylar Line in the Point *p*, which ſhall be the Foot of the *Style* required, whereof *p ſ* ſhall be the height.

But if we have not the *Center* of the *Dial*, we muſt draw the Line *S ſ* by the Point S which determines the Inclination of the *Axis* with the

Sub-

Subſtylar Line, by the Practice of the 16*th Chapter*; and we ſhall find as before the *Point p* for the Foot of the *Style* required, whereof the height ſhall be *p ſ*.

V.

Fig. 20. If after you have drawn the *Meridian Line* C M and the *Subſtylar Line* B P, we cannot have the *Equinoctial*, becauſe the *Style* has been put too long, we may diminiſh it as much as we pleaſe, without changing either the Foot thereof or the Subſtylar Line; but we muſt find another Meridian and another Horizontal Line which may anſwer to that *Style*, and theſe Meridian and Horizontal Lines ſhall be parallel to the firſt Meridian and Horizontal Lines.

Therefore draw the Line *e m* M by any Point of the Subſtylar Line as *e*, which may be perpendicular to it, that Line may be the Equinoctial Line; but the height of the Style muſt be changed in drawing *e ſ* perpendicular to the Line S *ſ* which determines the inclination of the *Axis* with the Subſtyle, and that Line *e ſ* meeting S P which is a perpendicular to the Subſtylar Line by the Foot of the *Style*, and which is his height, ſo that for the Equinoctial Line *e m*, P *z* ſhall be the height of the *Style* required; but there muſt be another *Meridian* found, whether the *Center* of the *Dial* be found or not found.

Take

Take on the *Subſtyle* the length *e* B equal to *e* draw B M, and alſo take *e b* on the Subſtyla Line equal to *e z*, and by the Point *b* draw *b n* parallel to B M meeting the *Equinoctial Lin* *em* in the Point *m*, and let *c m* be drawn paral lel to C M, the Line *c m* ſhall be the *Meridian* for the *Style*, whereof the Foot is in P, the heigh is P *z*, and the Equinoctial Line *e m*.

And we may yet change if we will the Foo of the *Style* or its height, according as neceſſity requires by the former Obſervarions.

But for the *Horizontal Line* D H which ha been found for the *Style* whoſe Foot was P an PS the height, the which meets the Subſtyle at the Point H, if you would find another for th *Style* whoſe height is *P z*, without placing effe ctually that *Style* to make uſe of the *Practic* taught in the *4th Chapter*, you have no more t do but to draw S H, and by the *Point z* the Lin Z *b* parallel to S H meeting the Subſtyle in *b* and the Line *d b* parallel to D H, ſhall be th *Horizontal Line* required.

VI.

Laſtly, a *Dial* being drawn on a *Plane*, we ma transfer it in what other place we will on the ſam *Plane*, by drawing of parallel Lines to thoſe tha are drawn, ſo that we keep the ſame order an the ſame proportion between them in their meet ings, but the *Style* ought to be put at the *Poin* which anſwers to the *Point* of the firſt, which i for its Foot.

THI

THE SECOND PART.

THE PREFACE.

Concerniug the choice which we ought to make of the Practices to draw the Subſtylar, Equinoctial, *and* Meridian Lines ; *and to place the* Center *of the* Dial , *according to the Expoſitions of the* Planes *propoſed.*

HAving taught in the firſt *Part* of this *Treatiſe* different *Practices* to find the principal Lines of *Sun Dials* , I think it will not be amiſs to give ſome Inſtructions concerning the Uſe which we may make of theſe Practices , following the different Expoſitions of the *Planes* propoſed on which the *Sun Dials* are to be drawn.

Therefore it is neceſſary to know ſomething near, the Poſition of the *Plane* in regard of North or South before we begin any thing ; which may

be

be done by a ſmall Declinatory, which preſently ſhews on what ſide the North, South, Eaſt, or Weſt is; which thoſe that are uſed to obſerve the *Sun*, may know by ſeeing in what manner it ſhines upon the Plane according to the Hour and Seaſon of the Year.

Then we may well conceive after what manner the *Axis* ſhall meet with that Surface, and conſequently we may judge of the Poſition of the *Subſtylar Line*, of the *Equinoctial*, and alſo of the whole *Dial*.

But conſidering a *Dial* wholly made, it is not difficult to know among divers manners which we may uſe, that may be moſt fit and moſt eaſie for the Conſtruction of the *Dial*.

Therefore we may preſently ſee that it will be uſeleſs to find the *Center* of a *Dial* or the *Meridian* on a *Plane* which comes near either to the Eaſt or Weſt, and that the Equinoctial Line being ſet on ſuch a *Plane*, we need not find the Point of Mid-day, and that we muſt uſe the Point of the Sixth Hour to begin the Diviſions of the Horary Intervals on that Line, or on the *Horizontal Line*.

Alſo we may ſee that on theſe ſorts of *Planes* we cannot uſe the Practices where we ought to have the Points of Shadows after Mid-day, which may be anſwerable to others taken in the Morning; for that which is of thoſe which give the Poſition of the *Meridian* of the *Plane*, which is the

the Subſtylar Line, by the correſpondent Points, we can never fitly make uſe of them on theſe ſorts of *Planes*: For if the firſt Point hath been mark'd a little too far from that Meridian, we can never have it's correſpondent Point.

Alſo we muſt not uſe the Practices of correſpondent Points of Shadows, or the Tract of the Shadow, if the *Circle* that is deſcribed from the Foot of the *Style* as a *Center*, meets that Tract in *Angles* too acute; for we cannot determine exactly *that meeting*, and this inconvenience may happen to the Practices on all ſorts of *Planes* in any Seaſon of the Year.

If the *Dial* be large, or if the *Declination* of the *Sun* has changed conſiderably between the *Obſervations* of the *Points* of Shadow, we have not exactly the Lines which we ſeek by thoſe Practices where we ſuppoſe that it hath not been changed between the *Obſervations*.

In the Practices of this *Second Part* we ſuppoſe always that the Equinoctial Line or Horizontal Line is drawn, and that we have mark'd on that Line the *Point* where the Hour of 12 or 6 meets with it; at which *Points* we begin the Diviſion of the Hours on thoſe *Lines*; but to draw them we muſt have the *Center* of the *Dial*, or at leaſt the *Inclination* of the *Axis* to the *Subſtylar Line*, which has been taught in the *Firſt Part*.

CHAP. I.

To mark the Points *of the* Aſtronomique Hours *on the* Equinoctial Line, *and by thoſe* Points *to draw the* Hour Lines.

Fig. 21. LEt S P be a *Style*, whereof S is the Point and P the Foot, E XII, is the Equinoctial Line; on which the Point XII is the meeting of the Equinoctial with the Meridian, and the Point VI is the meeting thereof with the Hour of Six and with the Horizon; P E A is the Subſtylar Line which meets with the Equinoctial in E.

Make EA on the Subſtyle equal to E S, which is the Diſtance between the *Point* E of the Equinoctial and S the *Point* of the *Style*, and draw A XII or A VI, or both of them, if you have thoſe *two Poinss* on the Equinoctial Line; the which two Lines A XII, A V I, ought to make a *Right Angle* at the *Point* A; then on the Point A as a *Center*, at any Diſtance deſcribe an Arch of a *Circle* *b c*, which ſhall cut the Lines A XII and A VI at the Points *b* and *c*, and divide the *Circle* from 15 to 15 Degrees, beginning at the

the *Point b*; or at the *Point* C; then draw ſtrait Lines from the *Center* A, and by the Points of the Diviſion of the Circle, which muſt be prolonged, if it be neceſſary, to the Equinoctial Line, on which it ſhall give the Diviſion of the Hours which are to be mark'd, according to the apparent motion of the *Sun* from Eaſt to Weſt.

Then by the *Center* of the *Dial*, and by the *Points of* the Hours which are mark'd upon the Equinoctial Line, draw ſtrait Lines, which ſhall be the Hour Lines.

But if we have not the *Center* of the *Dial*, and we have only the Inclination of the *Axis* Z *z* to the Subſtylar Line *e* E, you muſt take any *Point*, as *e*, on the Subſtylar Line, and by that *Point e* having drawn a ſtrait Line *e* 12, parallel to the Equinoctial Line E XII; and drawing *e z* perpendicular to Z*z*, make *e a* equal to *e z*, and by the *Point a* draw the ſtrait Lines *a* 11, *a* 12, *a* 1, *&c.* parallel to the Lines A XI, A XII, A I, *&c.* and by theſe Points, where theſe Lines meets with the Line *e* 12, and by thoſe which are correſpondent to them on the Equinoctial Line, draw the Hour Lines 11 XI, 12 XII, 1 I, 2 II, *&c.*

Demonſtration.

The Plane *S XII E is the* Plane *of the Equinoctial by the Conſtructions of the* Firſt Part, *and the Triangle A XII E being the ſame with the Triangle ſ XII E, we ought to put upon the* Plane *of*

 the

the Triangle A XII E *the Hour Lines* 15 *Degrees the one from the other about the Point A*, *in beginning from the Line A* XII *or A VI*, *which are the Interſections of the* Plane *of the Meridian*, *or of the* Plane *of the Hour Circle with the* Plane *of the Equinoctial*; *but all the* Planes *of the Hour Circles interſect one another in the* Axis, *and do alſo paſs by the* Center *of the* Dial *which is the meeting of the* Axis *with the* Plane *of the* Dial: *Therefore the ſtrait Lines drawn from the* Center *of the* Dial *to the Points of the Hours mark'd on the Equinoctial Line ſhall be the Hour Lines of the* Dial, *if the* Dial *has no* Center; *or if the* Center *be at a great diſtance from the Equinoctial Line*, *the Practice that is here taught for that Caſe gives the Points on the Line* e 12, *which is parallel to the Equinoctial Line*; *ſo as all the diſtances of the Hour Lines on that Line* e 12, *are in the ſame proportion with their Diſtances on the Equinoctial Line*, *and that each of theſe Diſtances on the Line* e 12 *have the ſame proportion to their correſpondent Diſtance on the Equinoctial Line*, *as the Line* e z *has to the Line* E Z; *therefore it follows that if the* Dial *have no Center all the Hour Lines are parallel to one another and to the* Axis. *And if it have a Center*, *all the Lines ſhall meet in that Center*: *which is very manifeſt*, *becauſe of the like Triangles which are made by the Lines* E XII, e 12, *which are parallel to one another.*

CHAP.

CHAP. II.

To mark the Points *of the* Aſtronomique Hours *on the* Horizontal Line. *And to draw the* Hour Line *by theſe* Points.

Fig. 22. S Is the Point of a *Style* given, whereof P is the Foot, M H D is the Horizontal Line, P H is drawn by the *Poine* P (the Foot of the *Style*) perpendicular to the Horizontal Line; M is that Point where the Meridian Line *interſects* the Horizontal Line, upon the Line H P ſet H ſ equal to H S, and draw the ſtrait Line ſ M *a*, and make the Angle M ſ A equal to the Angle of the elevation of the *Pole* above the *Horizon*, then from any *Point* as A, taken on the Line ſ A, raiſe a perpendicular from 12 to ſ A, till it meet with ſ M in 12, and draw the Line 9, 12, 4, perpendicular to ſ *a*, and make 12 *a* equal to 12 A; and from the *Point a*, as a *Center*, deſcribe a Circle at any Diſtance, and divide it into equal parts from 15 Degrees to 15 Degrees, beginning the Diviſion where the Line *a* 12 interſects the Circle; and draw Lines

from the *Point a* , to the Divisions of the Circle, to meet with the Line 9.4 at the Points 9, 10, 11, 12, 1, 2, 3, 4, *&c.* And by the same Points and the Point *ſ*, draw ſtrait Lines which ſhall meet the Horizontal Line in *the Points* of the Hours required , which muſt be mark'd according to the *Diurnal Motion* of the *Sun* , of which the *Point* M ſhall be Noon , and D the Point of the Hour of Six.

If the *Line ſ*D drawn perpendicular to *ſ*M meet with the *Horizontal Line* at the *Point* D, that *Point* ſhall be the Hour of Six on the *Horizontal Line* , which is the ſame Point where the *Equinoctial Line* ought to meet with the *Horizontal Line* , as hath been taught in the *Firſt Part.*

If we have not the *Point* of Mid-day on the *Horizontal Line* , and we have but D the *Point* of the Hour of Six , then draw *ſ*D and *ſM* perpendicular to *ſ*D , the which *ſ*M meets the *Horizontal Line* , or not in M , for it is indifferent ; then we do the ſame as we did before to find the *Points* of the Hours on the *Horizontal Line*. We may ſee by this *Practice* that it is not neceſſary that the Line *ſ*M *a* ſhould meet with the *Horizontal Line.*

The Hour Lines are to be drawn from C the *Center* of the *Dial* , and by the *Points* of the Hours which have been found on the *Horizontal Line.*

If the *Dial* have no *Center* , it is better to follow the Method of the precedent *Chapter* than

this,

this, ſeeing that we have always an Equinoctial Line in that Caſe.

Demonſtration.

The Plane *ſMD is the ſame with the* Plane *SMD by Conſtruction, which is the* Plane *of the Horizon; therefore all this Operation muſt be conſidered as made on the* Plane *of the Horizon which paſſes by the Point of the* Style *S, whereof ſM is the Meridian given, or the Subſtylar Line, or ſD is the Line of the Hour of Six, the Point ſ is the Center of the* Dial, *the Line ſ*A *is the Inclination of the Axis to the Subſtylar Line ſM; therefore it is that taking the Point A at pleaſure for the Point of the Style, by the Practice of the 11th Chapter the Line 9, 4, ſhall be the Equinoctial anſwering to the Point of the Style, and by the Practice of the former Chapter the Lines ſ12, ſ1, ſ2, &c. ſhall be the Hour Lines on that* Plane, *which meets the Horizontal Line MHD, which is alſo on the ſame* Plane *by Conſtruction with the Points of the Hours which they deſign; therefore the Lines which are drawn thro thoſe Points of the Hours on the Horizontal Line, and by* C *the Center of the* Dial, *ſhall be the Hour Lines of the propoſed* Plane.

CHAP.

CHAP. III.

Six Intervals of Hours following one another being given, to draw all the other Hours.

Fig. 23. AS let the Six Intervals of Hours from C A to C F be given, draw E *e* parallel to *c* 5, cutting *c* A in *the Point* A, C B in *the Point* B, C D in *the Point* D, *&c.* And make A *b* equal to A B, A *d* equal to A D, *&c.* And from the *Center* C, and through the *Points* *b d e*, &c. draw the Lines of the Hours that follow the precedent Hours.

If also you would have other Hours following the first or last found, you must repeat the Operation in drawing another Line as E *e* parallel to that which is the last of the Six Intervals of Hours.

If the *Dial* has no *Center*, you must draw another Line as *ſt* parallel to E *e*, on which find the *Points* of the Hours as you found them on the Line E *e*, and in joyning the *Horary Points* of the two parallel Lines E *e* and *ſt*, you shall have the Hour Lines required.

Demon-

Demonstration.

If we imagine a Plane *to pass by the Line* E *A* e, *and which is parallel to the* Plane *of the Hour Circle of the Line* C 5, *it is manifest that the* Plane *of the Hour Circle which passes by the Line* C *A*, *shall be perpendicular to the* Plane *that passes by* E *A* e, *because that the* Planes *of the Hour Circles* C *A*, C 5, *are inclined to one another by a Right Angle, and by consequence the Axis meets not with the* Plane *drawn by* E *A* e, *and it shall be parallel to it, and all the Lines which are the meetings of the* Planes *of the Hour Circles with the* Plane *that passes by* E *A* e, *shall be all parallel to one another and to the Axis.*

But because the Plane *that passes by* C *A is perpendicular to the* Plane *by* E *A* e, *the meetings of the Hour Lines upon that last* Plane *shall be equally distant by order from the meeting of the* Plane *drawn by* C *A; that is to say, that the first on one side shall be as far distant as the first on the other side, and that the second hour on one side shall be as much as the second hour on the other side, and so following in order; therefore the Line* E *A* e *which is on that* Plane, *meets the parallel Hour Line at equal distance by order, both on the one and the other side of the Point* A, *which was to be demonstrated.*

We

We may conceive the Plane *that passes by* E *A* e, *as the Plane of a* Dial *which has no Center, upon which all the Hour Lines are parallels to one another and to the* Axis; *and that the Hour Line which passes by the Point A is the Meridian or the Substylar Line of the* Plane; *then we may see manifestly that the Hour Lines that are on one and the other side of that which passes thro A, are equally distant from it in order, and by consequence if we draw a strait Line any way upon that* Plane, *the meetings of the Hour Lines both on the one side and on the other of that which passes by A, shall be at an equal Distance from A.*

CHAP. IV.

To draw the Parallels of the Twelve Signs.

Fig. 24. First draw the Line *sc* and *sa* at Right Angles to it at the Point *s*, make the Angles *asd*, *ask*, each 23 Degrees 30 Minutes, and the Angles *asf*, *asi*, each of 20 Degrees 11 Minutes, and the *Angles asg*, *ask*, each of 11 Degrees 30 Minutes, the Line *sa* denotes the Equinoctial, which is the beginning of *Aries* and of *Libra*; the Line *sk* denotes the be-

beginning of *Taurus* and *Virgo*, *ſi* the beginning of *Gemini* and *Leo*, *ſk* the beginning of *Cancer*, which is the *Tropick* of the ſame *Sign*; *ſg* the beginning of *Scorpio* and *Piſces*, *ſf* the beginning of *Sagittarius* and *Aquarius*, *ſd* the beginning of *Capricorn*, which is the *Tropick* of the ſame *Sign*.

If the *Center* of the *Dial* be towards the *North* in regard to the *Point* of the *Style*, make *ſc* equal to S C of the *Dial*, which is the diſtance between the *Point* of the *Style* and the *Center*, but if the *Center* be towards the *South*, in reſpect of the *Point* of the *Style*, make *ſc* upon *cſ* prolonged on the other ſide of the *Point* *ſ*.

Then to find the *Points* of the Parallels of the Signs upon the Hour Lines; as for *Example*, On the Line of Mid-day, you muſt take the diſtance S XII from the *Point* of the *Style* S to the *Point* XII, which is the *Interſection* of the Line of Mid-day with the Equinoctial, and ſet it from *ſ* to 12 upon the Line *ſa*, and having drawn the Line *c* 12 which cuts the Lines of the Signs in the *Points* *d*, *f*, *g*, *h*, *i*, *k*, then we tranſport the Intervals 12*h*, 12*i*, 12*k*, 12*g*, 12*f*, 12*d*, in XII H, XII I, XII K, XII G, XII F, XII D, on the one and other ſide of the Equinoctial Line, as they are on the two ſides of the Line *ſa*.

And after the ſame manner having found the other *Points* upon each Hour Line, and likewiſe on the halves and quarters, or other Lines coming

from

from the *Center*, we draw by all the *Points* which belong to the ſame Sign, the Line of the Parallel of the Sign, and ſo for each of them in particular.

But if we have not the Interſection of the Equinoctial Line upon the Hour Line, on which you would have the *Points* of the Signs, in that caſe you may have always the *Center* of the *Dial*; therefore it it is, that having taken any *Point*, as R on the Hour Line, and having made the Triangle *c ſ r* on the Baſe *c ſ* equal, and like to the Triangle C R S on the Baſe C S, and the Line *c r* being continued if it be neceſſary, ſhall meet with the Lines of the Signs in thoſe Points, which being ſet on the *Dial*, in obſerving from the *Point* C which is the *Center*, the ſame Diſtances which they have from the *Point c.*

But if we have not the *Center* of the *Dial*, we may always have the Equinoctial Line; therefore, having taken (as for *Example*) the third Hour, on which we would have the Points of the Parallels of the Signs, then having taken the *Point* R at pleaſure, and having mark'd *ſ* 3 on the Line *ſ a* equal to S III, which is the diſtance between the *Point of the Style* S, and the *Point* where the Line of the third Hour propoſed interſects the Equinoctial Line; on that Line *ſ* 3 for the Baſe let the *Triangle ſ* 3 *r* be made equal and like to the *Triangle* S III R, which has *S* III for its Baſe; and draw *r* 3 prolonged, which ſhall interſect the Lines of the Signs in *Points* which are to be

tranſ-

transfered to the Line of the Third Hour, as we have taught here before.

The Construction of this Practice is so plain, that it needs no Demonstration, for it is easie to see that the *Plane c ſ* 12 is the same with the *Plane* of the *Hour Circle* C S XII, and so of the rest.

A part of an Arch of a Sign being given, which is Parallel to the Equator, we may describe that Arch by the Practice of the following *Chapter*.

CHAP. V.

The Equinoctial Line being given, we may draw a Parallel to it by a Point given on an Hour Line.

Fig. 25. THe Equinoctial Line A G is given, and the Hour Lines *a* A, P B, *c* C, *d* D, *&c.* which intersect the Equinoctial in the *Points* A B C D, *&c.* and the *Point* P on one of the Hour Lines is also given, by which it is required to draw a Parallel to the Equator.

By the *Point* P, and by the *Point* A, on the Equinoctial, the Point of the Hour next to B, on which is the given Point P, draw A P meeting with

with the next Hour to P B at the Point *c*, then draw *c* E, ſo as the Points E and A may be equally diſtant from the Hour *c* C; the interſection *d* of the Line *c* E with the Hour Line D *d*, ſhall be one of the Points of the required Parallel; after the ſame manner we find another *Point* *f*, and ſo of the reſt.

But here it is to be obſerved, that if there be only whole Hours on the *Dial*, we ſhall not have the Points of the Parallel but for every other Hour; and if we have the Lines of the half Hours, we ſhall have the Points of the Parallel from Hour to Hour, and if we have the Lines of the Quarter of Hours, we ſhall have the Points of the Parallel from Half Hour to Half Hour, and ſo of the reſt; for if A P meet the Line of the half Hour *ſb* at the Point *ſ*, having drawn *ſ* D ſo as *b* D and *b* A may be equal, *ſ* D ſhall meet the Hour Line *c* C at the Point K, which is as far from *ſb* as P B, the Point K ſhall be one of the Points of the Parallel.

If inſtead of the Equinoctial Line we have one of its Parallels, that parallel being given, we do the ſame thing as if it were the Equinoctial Line.

Demon-

Demonstration.

The strait Lines on the Plane of the Dial *represent great Circles of the Sphere, therefore it is that the two great Circles represented by* C *A and by* C E, *which are equally inclined to the Hour Circle* c C, *because that* C *A and* C E *are equal, meeting P B* and *D* d, *which are equally distant and equally inclined to* C c, *in the Points* P *and* d, *equally distant from the Equinoctial Line B D, therefore* P *and* d *shall be in one and the same parallel to the Equator, and so of the other Points.*

CHAP. VI.

To draw the Italian *and* Babilonian *Hours upon an* Horizontal Plane.

Fig. 26. THe AstronomiqueHours being drawn on the *Dial* whose *Center* is C, and the *Meridian* C A and V A the *Equinoctial Line*, c E being divided into two equal parts in A, let the Points *b*, *c*, *d*, *e*, *f*, *g*, *h*, &c. of a Parallel to the Equator, be found on the Hour Lines by the Practice of the fore-going Chapter, which Parallel passes by the *Point* A.

The

The Line A 12 parallel to the Equinoctial, ſhall be the Line of the 12th *Italian Hour.*

The ſtrait Line *b* VII, which paſſes by the Point of the Seventh Hour in the Morning of the Equinoctial, and by the *Point* of the Firſt Hour after Noon of the Parallel, ſhall be the Line of the 13th *Italian Hour.*

The ſtrait Line *c* VIII which paſſes by the Point of the Eighth Hour in the Morning on the Equinoctial Line, and by the Point of the ſecond Hour after Noon of the Parallel ſhall be the 14th *Italian Hour*, the ſtrait Line *d* IX, which paſſes by the Point of Nine in the Forenoon on the Equinoctial, and by the Point of Three in the Afternoon on the Parallel, ſhall be the 15th *Italian Hour*, and ſo of the reſt; there being always Six Hours diſtance between the Hour of the Equinoctial and that of the Parallel.

The *Babylonian Hours* are mark'd after the ſame manner, but only that which is done on one ſide of the Meridian for the *Italian Hours*, is made on the other ſide of the Meridian for the *Babylonian Hours*, and they are counted after another manner; as for *Example*, the ſtrait Line that paſſes by the Point of Mid-day of the Equator, and by the Point of the Sixth Hour in the Morning of the Parallel, is the Sixth *Babylonian Hour*; that which paſſes by the firſt Hour Afternoon on the Equinoctial, and by the *Point* of the Seventh Hour in the Morning on the Parallel, ſhall be the Seventh *Babylonian Hour*, and

ſo

ſo following, ſo as A 12 parallel to the Equator ſhall be the 12th *Babylonian Hour* for the Horizontal *Dial.*

Demonſtration.

Fig. 27. *If we imagine a Parallel to the Equator, which paſſes thro the Interſection of the Meridian and Equator, it is evident that the Horizontal Circle ſhall touch that parallel at the Point where the Meridian Circle cuts it; and if we conceive that the Horizontal Circle be faſtened with the Parallel and the Meridian, then when the Meridian hath advanced 15 Degrees on the Equator in going according to the Motion from the Eaſt to the Weſt, the* Plane *of the Horizontal Circle in that Poſition, ſhall mark the firſt* Italian Hour *on the Weſt part, and the firſt* Babylonian Hour *on the Eaſt part; and then when it hath advanced 15 degrees more, the* Plane *of the Horizontal Circle ſhall mark the ſecond* Italian ~~*and*~~ *and* Babylonian *Hour, and ſo on; ſo as then when it is arrived to the 12th Hour, the Meridian Circle ſhall be found in its firſt Poſition, for it has made half a Revolution; but the Horizon that cuts the Plane of the Meridian at Right Angles, ſhall be found in a Poſition oppoſite to that which it had at firſt; that is to ſay, that if S H be the meeting of the Horizon with the Meridian, S C the Axis, C E the* Plane *of the* Dial *parallel to the Horizon; after that the Meridian hath paſſed over 12 Hours of the Equator, or that S H*

has passed over 12 Hours of the Parallel; the Line S H shall fall in 12 S A: So as the Angle 12 S C shall be equal to the Angle H S C, therefore C S A shall be an Equicrural Triangle; and so also the Triangle S A E shall be Equicrural, S E being the meeting of the Equator and Meridian; therefore S A is equal to A C and A E both together; therefore C A and A E are equal: But C E represents the Meridian of the Horizontal Dial, wherein C is the Center, E the Intersection of the Equinoctial, and A the Intersection of the 12th Italian *or* Babylonian, *which must be perpendicular to the Meridian; for in that Position the* Plane *of the Meridian is return'd to its place, and the Horizon which has not changed its Inclination which it has with the Meridian remains always at Right Angles to it. Therefore it shall intersect the* Plane *of the* Dial *in a Line perpendicular to the Meridian, the* Plane *of the* Dial *being parallel to the Horizon in that first Position, and that Line ought to pass thro the Point A of the Meridian, which divides the Line C E into two equal parts.*

If we conceive a Cone which has for its Base the Parallel to the Equator H 12, and for its Vertex the Center of the Sphere, *the Section of that Cone with the* Plane *of the* Dial, *shall be the representation of that Parallel, but the* Plane *of the* Dial *is parallel to one of the* Planes, *which touches the Cone; therefore the Section or the Representation of that Parallel shall be a* Parabola.

And

And seeing that all the Planes *of the* Italian *and* Babylonian *Hours are also the* Planes *which touch that Cone, the Lines of those Hours shall touch the Parabola which is in that Section.*

CHAP. VII.

To draw the Italian *and* Babylonian Hours, *on a* Plane *which is not* Horizontal.

Fig. 28. THe Astronomique Hours being described, and the Horizon R H which is one of the Hours required, being drawn on the *Plane* of the *Dial* with the Equinoctial Line, we draw a Parallel to the Equator *d*, *b*, *R*, *e*, *f*, *g*, which passes by R the Intersection of the Horizon, with any Hour Line, as in this *Example*, with the Hour Line of II, by the *Practice* of the 5th *Chapter*. And seeing that the Horizon, which is the Line of the 24th *Italian Hour*, intersects the Parallel in R, at the Point of the Second Hour after Noon, and the Equinoctial Line at the Point of the Sixth hour after noon the Line of the first *Italian Hour* shall pass by the Point *e* of the Parallel, which is the Third Hour after Noon, and by the Point of the Equinoctial; the Line of the Second *Italian Hour* shall pass by the Point *f* of the

Parallel, which is the Fourth Hour, and by the Point of the Eigth Hour on the Equinoctial Line; and ſo of the reſt we find all the Points by which the *Italian Hours* ought to paſs, ſo as the Eighteenth *Italian* Hour paſſes always by the Point of Mid-day of the Equinoctial Line, and by a Point of the Hour of a Parallel, which ſhall be ſo far from the Point of Mid-day as the Point R of the ſame Parallel, which is the Interſection of it with the Equinoctial Line, is the Point of the Sixth Hour after Noon, which is in this *Example* at Four Hours diſtance, ſeeing the Point R is an Hour of the Afternoon.

Fig. 29. But if the Point R, by which the Parallel to the Equator is deſcribed, were the Interſection of an Hour before Noon, we muſt conſider that that Parallel ought to meet alſo the Horizon in a Point of an Hour, which is ſo far from Noon, as that is by which we have deſcribed it; for *Example*, If the Point R were the Interſection of Nine in the Morning with the Horizontal Line, the Parallel to the Equator deſcribed by the Point R, ought to meet the Horizontal Line in the Point H, which is upon an Hour Line, ſo far diſtant from Noon as is the Point R; that is to ſay, that the Point H ſhall be the meeting of the Third Hour after Noon with the Horizontal Line; and the Line of the 24th Italian Hour, which is an Occidental Portion of the Horizon, ought to be taken from the Point of the Third Hour of the

the Parallel with the Point of the Sixth Hour afte Noon of the Equator, and in counting as we have done before, we ſhall find that the Firſt Italian Hour ſhall paſs by the Point of the Fourth Hour on the Parallel, and by the Point of the Seventh Hour after Noon on the Equinoctial; and that the Line of the Second Italian Hour ſhall paſs by the Point of the Fifth Hour of the Parallel, and by the Point of the Eighth Hour of the Equinoctial, and ſo on; and we draw only thoſe that are viſible, for the others are of no uſe, and ſerve only to count and to place thoſe which are of uſe.

Theſe Rules are for the Italian Hours, but for the Babylonian Hours, which have for the Twenty fourth Hour the Oriental Part of the Horizon, if the Parallel which is deſcribed by the Point R of the Horizon, were the meeting of the Horizon with the Line of the Ninth Hour before Noon, the Firſt Babylonian Hour ſhall paſs by the Point of the 10th Hour in the Morning of the Parallel, and by the Point of the 7th Hour in the Morning on the Equinoctial; the Line of the Second Babylonian Hour ſhall paſs by the Point of Eleven before Noon on the Parallel, and by the Point of Eight on the Equinoctial, and ſo of the reſt; and if the Point R of the Parallel were the Point of any Afternoon Hour, we muſt take its correſpondent before Noon to begin to count the Babylonian Hours, which is the contrary of that

which we have done for the Italian Hours.

We muſt obſerve, that if we deſcribe the Lines by the Interſections of the Aſtronomique Hour-Lines with the *Italian* or *Babylonian*, in taking thoſe Points of Interſection in order from the Equinoctial Line, thoſe Lines ſhall be the Parallels to the Equator, which ſhall meet the Horizontal Line in the ſame Points where the *Hour Lines* meet.

Demonſtration.

After that which hath been demonſtrated in the afore-going Chapter, concerning the generation of the Italian and Babylonian Hours, it ſuffices here to explain after what manner the Horizontal Line meets the Aſtronomique and Italian Hour Lines in one and the ſame Point.

It is eaſie to ſee that when the Horizon, which we have ſuppoſed to be faſtened to the Meridian without changing its Inclination that it has with it, is moved by the motion of the Meridian, that if the Meridian advanceth 15 degrees on the Equator; alſo that Point of the Horizon, let it be what it will be, ſhall advance 15 degrees on the Parallel which that Point deſcribes: Therefore if we deſcribe a Parallel to the Equator by one Point of the Horizon, by which Point paſſes an Aſtronomique Hour Line, it is manifeſt that all the ſame Hour Lines ſhall divide that Parallel from 15 to 15 degrees; Therefore it is that all the Italian Hour Lines

which

which ought to paſs by the Points of the Hours of the Equinoctial, ſhall alſo paſs by the Points of the Hours of the Parallel; and it is alſo manifeſt that we may draw Six Parallels to the Equator by the ſix Interſections of the Horizon with the ſix Hour Lines 12, 1, 2, 3, 4, 5, *and that the Italian Hours paſs in order by the Points of the Hours of thoſe Parallels; and it alſo follows that all the Lines of the Italian and Babylonian Hours ſhall touch the Parallel deſcribed by the Point of meeting of the Horizon and Meridian, and that all the Points of touching ſhall be the meeting of the Aſtronomique Hours with the Parallel, and that there will be always ſix Hours Interval between the touching Point upon the Parallel, and the Point of meeting of the ſame Hour upon the Equator; for the* Plane *of the Horizon touches that Parallel at the Point of Mid-day, the which Point deſcribes the Parallel.*

CHAP. VIII.

To continue the Description of the Italian and Babylonian Hours, when the Parallel or the Equator is wanting on the Plane *of the* Dial.

Fig. 28. IF the Point *b* be the last that is found on the Parallel by means of the Equinoctial Line, following the Practice of the *5th Chapter* of this *Second Part*, and that the Line *b* III be the last *Italian Hour* which we can mark by help of that Parallel, that Line *b* III shall meet with some Astronomical Hour in some Point as *m*, if by the Practice of the *Fifth Chapter* of this Part we find the Points *l n o* of the Parallel which passes by *m*, and if they be on the Hour before or after that, on which is the Point *m*; we continue to draw the Lines of the *Italian* or *Babylonian Hours* by the Points of the Hours of the Parallel *m n o*, and by the Points of the Hours of the Equator, in following the same order as before, and if the Equator be wanting in using the Practice taught at the end of the *5th Chapter*, we shall find the Points of another Parallel by the Parallel that is given,

given; and then we may joyn the Points of the Hours on the two Parallels in following the former Order.

There is nothing in all this that deserves any long Explication, nor any other Demonstration, than that we have given in the Two fore-going Chapters.

CHAP. IX.

Four Astronomique Hours *being given, following one another in order, with the* Equinoctial Line.
To find the other Hours.

Fig. 30. LEt the four Hour Lines following one another be A *a*, B *b*, C *c*, D *d*, with the Equinoctial Line E F; from a Point *a*, taken at pleasure in one of the last Lines A *a*, having drawn *a* D which cuts B *b* in B, and C *c* in *g*; also by the same Point *a*, having drawn *a* C which cuts B *b* in *h*, let A *h* be drawn, which meets C *c* in *c*, and B *g* which meets D *d* in *d*: let *c b*, *c d* be drawn prolonged to the Equinoctial Line to the Points E, F, the Hour Lines E *e*, F *f*, drawn by the Points E F, shall be the Hour

Lines

Lines required ; whereof E *e* ſhall be diſtant from A *a* one Hour, and F *f* ſhall be two Hours from D *d* : Therefore B D being prolonged to *f* in the Line F *f*, and F *d* to *l*, in the Line B *b*, having drawn C *f* which cuts D *d* in *i*, *l i* prolonged ſhall meet the Equinoctial in M, by which the Hour Line M *m* ſhall be between the two Hour Lines D *d* and F *f*, and theſe *Seven* Hour Lines being found, we may have all the reſt by the Practice of the *Third Chapter*.

Fig. 31. There are many Caſes where Three Hour Lines are ſufficient with the Equinoctial and Horizon ; for *Example*, If we have three Hour Lines *a* 2, *b* 3, *and* *c* 4, and the Equinoctial Line 24, and Horizontal Line *a c*, having drawn *a* 4, which cuts *b* 3 in *d*, and having drawn 2 *d* which cuts *c* 4 in *f*, draw 2 *c*, which cuts *b* 3 in *e*, and 4 *e*, which cuts *a* 2 in *h*; there ought a ſtrait Line to paſs by the three Points *f b h*, which ſhall meet the Equinoctial in the Point *g*, which is one Point of the Hour as far from the Hour *b* 3, as is the Hour Line of Six: Therefore if the Hour Line *b e* be the fourth Hour, *a h* ſhall be the third, and *g i* ſhall be the ſecond ; but in this *Example*, *b e* being the third Hour, *g i* ſhall be the 12th Hour.

The firſt Hour between 12 and 2 is found by drawing *g c*, which cuts *a* 2 in *k*, and 4 *k* which cuts the Hour Line *g i*, which was drawn by the Point *g* to the Point *i*, and in drawing 2 *i*, which

which cuts $g\,h$ in n, the Hour Line by the *Point* n shall be the first Hour.

Demonstration.

By that which hath been demonstrated in the 5th Chapter it is manifest that the Points a *and* c *are in the same Parallel; also that the Points* b *and* d *are also in the same; therefore the great Circles that are represented by the Lines c b* E, *c d* F, *are equally inclined to the Equator, the one on one side, and the other on the other, but c b* F *has the same Inclination as* a b *D, because of the equal parts of their Parallels* a c, b d; *and there is a space of three Hours between* c C *and F, and between* c C *and* E; *therefore the Hour which passes by the Point* E *is the next to that which passes by the Point A, but there are two between the Points D and F.*

By the same Construction the Points l *and* f *are in the same Parallel, but* f i c *and* l i *M represent great Circles equally inclined to the Equator; because of the Points* l *and* f *which are in the same Parallel, and equally distant from the Point* i; *therefore D C and D M represent equal parts of the Equator, which are each of them an Hours distance from one another.*

As concerning that which is of three Hour Lines, seeing that by Construction the Points a *and* f *are in the same Parallel, and likewise* h *and* c, *the Lines* f h *and* a c *which is the Horizon, represent Circles equally inclined to the Equator; therefore there*

is

is as much distance between the Point of the Hour of Six upon the Equinoctial and the Hour Line b e, *as there is between the same* Hour Line b e *and the Point* g. *We demonstrate as before that the Point* n *divides the interval between the* Hour Lines g i *and* a k.

CHAP. X.

A Dial *being given which is already drawn, to find the foot of the* Style *which did serve to draw it, and to determine the heighth thereof.*

Fig. 32. THe Line A B is the Equinoctial Line, and the Distance A B on that Line is the Interval of any six Hours; having divided A B into two equal parts in the *Point* G, from the *Point* G as a Center, on the Diameter A B describe the Circle A, S, B, *d*, *f*, and mark the *Points d* and *f* which divide the Semicircle into three equal parts; A F, F D and D B are each the Interval of two Hours on the Equinoctial Line: the Lines *d* D, *f* F ought to meet the Circumference of the Circle at the *Point* S, and the Line S E P drawn perpendicular to the Equinoctial Line shall be the Substyle.

If we have C the Center of the *Dial*, on the Diameter C E, having described the Semi-circle C Z E, and having applied in it the Line E Z equal to E S: Z P being drawn perpendicular to the Subſtylar Line E P, and meeting it at the *Point* P, that *Point* ſhall be the Foot of the Style, whereof P Z ſhall be the heighth.

But if we have not the Center of the *Dial*, having drawn *a e* parallel to the Equinoctial, and from the *Point a* having drawn *a ſ* parallel to A S which meets the Subſtylar Line in *ſ*, from the *Point* E as a Center, and Semi-diameter E S, deſcribe the Arch *z*, and from the *Point e* as a Center and Semi-diameter *e ſ* deſcribe the Arch *x*, and draw the ſtrait Line *x z* which ſhall touch the two Arches *x* and *z*, and that Line *x z* ſhall determine the Inclination of the *Axis* to the Subſtylar Line, and having drawn E *z* perpendicular to *x z* from the Point E, and from the Point *z* the ſtrait Line *z* P perpendicular to the Subſtylar Line E P, the Point P ſhall be the foot of the Style, whereof P Z ſhall be the height.

Demonſtration.

If we ſuppoſe the Circle A S B d *to be on the Plane of the Equinoctial, it is manifeſt that the Point of the Style ought to be one of the Points of the Circumference of the Semicircle A S B, be-*

cauſe

cause the six Intervals of Hours that are on the Equinoctial from A to B, for the Angle A S B, in what place soever the Point S is, ought to be a Right Angle: And also the Lines drawn from that Point S to the Points of the Hours of the Equinoctial Line, ought to comprehend equal Angles each of 15 *degrees in the same Point S; therefore the Arches A* f, f d, *and* d *B being each of* 60 *degrees, shall subtend Angles of* 30 *degrees, which are each in value two Hours, and consequently the Lines* f *F and* d *D shall be Hour Lines on the Equinoctial; therefore they meet in the same Point S upon the Circumference of the Circle which determines the Point of the Style in respect of the Equinoctial, and the Line S E drawn perpendicular to the Equinoctial Line, doth there give the Point E which is one of the Points of the Substylar Line; but the Substylar Line ought to be perpendicular to the Equinoctial Line; therefore the same strait Line S E P shall be the Substylar.*

The Axis ought to make a Right Angle with the Plane of the Equinoctial, therefore the Line E S which is equal to that which ought to meet the Axis on the Plane of the Equinoctial, ought to make a Right Angle with the Axis.

Therefore if we have c *the Center of the* Dial, *and if we describe the Semi-circle* C Z E, *applying* E Z *equal to* E S *in that Semi-circle, it is manifest that* C Z E *shall be a Right Angle, and that the Line* C Z *represents the Inclination of the Axis to the Substylar Line* C E, *and that the Point* Z *represents the*

the Point of the Style which has ſerved to draw the Dial: *Therefore* Z P *which is drawn perpendicular to* C E, *ſhall be the height of the Style, whereof the Point* P *ſhall be the foot.*

But if we have not the Center of the Dial, *it is manifeſt by Conſtruction that the two Lines* E S e ſ *having one of their extreams in the Points* E e, *the other will be had in the Axis, if the Line that joyns the two laſt extreams be perpendicular to thoſe two Lines which ought to be parallel; but it is alſo evident that the Line* x z *which touches the two Arches of Circles which have for their Semi-diameters* E S, e ſ, *ſhall be perpendicular to the two ſtrait Lines* E z, e x, *which come from the Center to the touching Points; therefore the Line* x z *gives the Inclination of the Axis to the Subſtylar Line, and the Line* z P *drawn perpendicular to the Subſtyle, ſhall determine the height of the Style, whereof* P *ſhall be the foot.*

CHAP.

CHAP. XI.

To place the Axis.

IF we would have the Hours ſhewn only by the ſhadow of the Point of the Style, we ought to make and to place the Style after ſuch a manner as may ſerve without changing of it.

We may give it divers forms, but one of the beſt is, to make it waved to the end, that the Shadow thereof may not unite with the Hour Lines in any place, and that we may always know that it is but only the Shadow of the Point that ſerves to ſhew the Hours.

But if you would have a portion of the *Axis* to ſhew the Hours, and that the *Axis* be repreſented by an Iron Rod, the Style we have placed ought to have the Point very ſmall, that it may enter into a little hole made in the Rod, ſo as the Point of the Style may exactly anſwer to the middle of the thickneſs of the Rod; the Style may remain, if you would have it, to ſupport the *Axis*; but if the *Axis* be not very long, and if it be ſtrong enough to ſuſtain it ſelf alone being faſtened

faſtened at one end, we may take away the *Style* when the *Axis* is fixed on the Surface of the *Dial*. We may do the ſame, if we faſten to the end of the *Style* a Point of an *Iron Wyer*, which there may be very ſmall, and may take away but the half of the thickneſs of the Rod, ſo as the *Dial* being drawn to that *Point*, there is nothing to be done but to take it away to place the *Axis*, whereof the middle of the thickneſs ought to anſwer to that Point; therefore whether the *Style* remains to uphold the *Axis*, or whether we take it away when the *Axis* is fix'd in its place, we muſt faſten it to the end of the *Style* to ſtay the end thereof, which ought to anſwer to the Center of the *Dial* if it have any.

Fig. 33. We may make the *Rod* which ſerves for the *Axis* as it is marked in the Figure, ſo as the hole ſignified by A may be made to lodge the Point of the *Style*, and that it be let in as far as the middle of the thickneſs of the Rod, the Point B which anſwers alſo to the middle of the *Rod*, ought to be applied exactly to the Center of the *Dial*, this Rod being ſo ſtayed at the Point B and at the Point A, we muſt faſten the Foot C on the *Plane* of the *Dial*.

But if you would not have a Foot to the *Axis* as G, and that you would only fix the Rod to the Center of the *Dial*, you muſt draw divers Lines which may paſs by the Center of

the *Dial*, and ſtay the Rod on the Point of the *Style* A, and by any other place, ſo as the end may enter in a hole made in the *Plane* of the *Dial* at the place of the Center, may be divided by the middle of its thickneſs by each Line that paſſes by the Center.

Fig. 34. Alſo we may make uſe of a thin Plate which muſt be cut according to the Inclination of the *Axis* with the Subſtylar Line; it muſt be ſet perpendicularly on the *Plane* of the *Dial*, in applying one of its ſides to the Subſtylar Line, and the other ſide paſſing by the *Point* of the *Style* ſhall ſerve for the *Axis*. If you would draw on the *Plate* a perpendicular Line to that ſide of it which is applied to the Subſtylar Line, and equal to the height of the *Style*, the which ſhall repreſent the *Style*, ſo as the end of that Line meet with the other ſide of the *Plate*, that Line when the Plate ſhall be ſet in its place, ſhall anſwer to the *Style*. See here divers Figures of theſe ſorts of *Plates* with their feet to faſten them on the *Plane* of the *Dial*.

CHAP.

CHAP. XII.

To Draw Dials *by Reflection.*

TO make a *Dial that* may ſhew the Hours by the Reflection of the Light of the *Sun*, you muſt make uſe of a ſmall piece of poliſhed Metal very even and flat; of a round form, and of abont an Eighth part of an Inch in Diameter; and having placed it; and faſtened it in a place very ſtable and immovable, we mark the Points of Light on the *Plane* where we intend to draw the *Dial*, which ſerve inſtead of the *Points* of Shadow, the middle of the Mirror or Glaſs ought to be conſidered as the Point of a *Style*, whereof we find the foot in drawing from the middle of the Glaſs a Line perpendicular to the Plane of the Dial, the Point where this Line meets with the Plane of the Dial, ſhall be the Foot of the Style.

We may find the Subſtylar Line, the Equinoctial Line, the Center of the Dial and Meridian by the practices, where we make no uſe of the Horizontal Line, nor of the height of the Pole.

Having found the Equinoctial Line, and the Point where the Meridian Line interſects it, we draw the Hours, following the Methods of the Second Part of this Treatiſe.

Here it is to obſerved that if the Inclination of the Glaſs be never ſo little changed, all the *Dial* will be conſiderably changed. Therefore theſe ſorts of *Dials* do very hardly remain many years in a good condition; for there always happens ſome alterations to the Wall on which they are fixt.

But if in place of the Glaſs we fill ſome ſmall Veſſel either of Glaſs or Potters Earth, of about an Inch in Diameter, with Water or Quick Silver, that Veſſel being put upon a place marked on ſome Tranſum of a Window or the like, ſo as you may always ſet it in the ſame place again, if you would take it away, the Reflection of the Light from the Water or Quick Silver, ſhall give the Hours on the *Dial* of which we muſt draw the Lines, as has been taught in the Firſt and Second Part of this Work, in obſerving only that the middle of the *Superficies* of the Water or Quick Silver, ſerves for the Point of the *Style*, and that the Operations which are made on the Horizon below for the *Dials*, which gives the Hour by the Shadow of a Point, ought to be made on the Horizon above, and that which is made above in thoſe *Dials*, in theſe *Dials* to be made below.

CHAP.

CHAP. XIII.

Concerning the Table *of the* Suns *Declination, and of those of the Difference of Meridians of divers considerable places in respect of* Paris.

THe *Tables* of the Declination of the *Sun* which are at the end of this Work, is calculated for the Meridian of *Paris*, and for each day at Noon, on the ſide are the Differences between the Declination of one day and that of the next day following; they are made for Four Years following one another. The Firſt Year begins in 1681, which is the Firſt after the *Biſſextile* or *Leap Year*, the Second is the following Year 1682, the Third is 1683, and the Fourth Year 1684, is *Biſſextile* or *Leap Year*.

Then afterwards there follows a *Table* of the Differences of Meridians of the principal places of the Earth in reſpect of *Paris*; it is calculated in Hours and Minutes, and it ſerves to find the Declination of the *Sun* in all thoſe places at any Hour propoſed. There is alſo another *Table* which is joyned to that, in which we may find

the Latitnde or Height of the Pole of the same places.

We begin the Days in these Tables and in the following Calculations, from each Day at Noon, and continue it to the next following Day at Noon, and we count the Hours to 24, soas the Third Hour in the Morning of any day proposed, is the Fifteenth Hour after Noon of the preceding day.

RULE I.

To continue the Tables *of the* Suns *Declination.*

IF you would have the Declination of the *Sun* for the Years following those that are calculated, you must add to the Declination increasing of each day, and substract from the Declination decreasing after Four Years fully past, one Minute for 32 Minutes of Difference, half a Minute for 16 Minutes of Difference, a quarter of a Minute for 8 Minutes of Difference, and so of the rest in proporti on.

Exam-

Example.

If you would know the Declination of the *Sun* for the *5th* of *January* at Noon for the Year 1685, which is the First Year after the *Bissextile* or *Leap Year*, and is the Fourth that follows to the Year 1681, whereof we have the Calculation in the Table. I find that the Difference between the *5th* and *6th* of *January* 1681, is 12 Minutes, and the Declination is decreasing; therefore I see that there must be substracted about $\frac{1}{3}$ of a Minute from the Declination of the *5th* of *January* 1681, the which is 20 Degrees 58 Minutes; and we shall have 20 degrees 57 Minutes $\frac{1}{3}$ for the Declination of the *Sun* the *5th* of *January* at Noon in the Year 1685; if there be Eight Years passed between the Year in which you would have the *Suns* Declination, and that which answers to it in this Table, that is to say, that which is equally distant from the *Leap Year*, you must double this Correction; if it be 12 Years, you must triple the Correction, and so forth.

RULE II.

To find the Declination of the Sun *at all Hours of the Day.*

HAving found in the Table of differences of the *Suns* Declination, the difference of the Declination of the *Sun* between the given Day and the next following Day, take the part proportional of that Difference anſwerable to the given Hours, which we add to the Declination of the ſame Day if it increaſe, but ſubſtract it if it decreaſe.

Example.

As to know the Declination of the *Sun* on the 15*th* day of *March* at 4 a Clock in the Afternoon for the Year 1683, I find in the Table that the Difference between the 15*th* and 16*th* day is 23 minutes and $\frac{1}{2}$ (for there is ſometimes 24 minutes and ſometimes 23 minutes) and the Declination of the *Sun* for the 15*th* of *March* at Noon in the Year propoſed 1683, being 1 degree 59 minutes, to which here muſt be added the proportional

tional part of the Difference for 4 Hours, which is the ſixth part of 24 Hours; therefore you muſt take the ſixth part of 23 minutes and $\frac{1}{2}$, which is about 4 minutes, which being added to the Declination found 1 degree 59 minutes, becauſe the Declination increaſes, and we ſhall have 2 degrees 3 minutes, the Declination increaſing on that Day at the required Hour; but if the Declination were decreaſing, we muſt ſubſtract the part proportional of the Declination found in the Table.

This ought to be underſtood of the Declinations of the *Sun* for the Meridian of *Paris*, as we find them in this Table, but for other places on the Earth, they are to be reduced according to the following *Rule*.

RULE III.

To know the Declination of the Sun *at a given Hour in any place that is ſet down in the Table.*

THe Table of Differences of Meridians of places which are here ſet down in reſpect of *Paris*, with the word *add* or *ſubſtract* ſhews how

how much later or ſooner it is Noon at *Paris* than at that place; the chief uſe whereof is to know the Declination of the *Sun* at a given Hour in any place ſet down in the Table.

Therefore if you would know the Declination of the *Sun* at a given Hour in any place that is in the Table, you muſt find the Difference of its Meridian in reſpect of *Paris*, and joyn it to the given Hour, if there be *add* after it, and if the Sum exceed 24 Hours, we take the overplus in the next following day to that as was given; but if there be found *ſubſtract*, we take that Difference of Meridians from the Hour propoſed; but if the Hour be too little, add 24 Hours to it, and then ſubſtract, and the remainer ſhall be attributed to the fore-going Day.

Having therefore found this Sum or Difference of Hours, we find the Declination of the *Sun* for that Hour at *Paris* by the Second Rule, and you ſhall have that which you require for the given place.

Example.

If we would know for *Rome* the Declination of the *Sun* for the 22th day of *Auguſt* 1682, at 4 a Clock in the Morning.

Firſt, Becauſe the Hour propoſed is in the Morning, I reduce it to 16 Hours after Noon on the 21th day of *Auguſt*, and having found in the

the Table of Differences of Meridians, that for *Rome* I muſt ſubſtract 47 minutes, therefore I take 47 minutes from 16 hours, and there remains 15 hours and 13 minutes after Mid-day on the 21th of *Auguſt* 1682; and by the Second Rule I find that the Declination for that day and that Hour is at *Paris* 8 degrees and 15 minutes, which is that which was required for 4 a Clock in the Morning the 22th day of *Auguſt* 1682 at *Rome*.

Another Example.

If you would know for *Peking* in *China* the Declination of the Sun at 2 a Clock in the Morning on the 25th of *July* 1684.

Firſt, becauſe the Hour propoſed is before Noon, I reduce it to the fore-going day, which will be the 24th of *July* at 14 Hours after Noon, and I find in the Table of Differences of Meridians, that for *Peking* I muſt ſubſtract 7 hours 45 minutes, the which being ſubſtracted from 14 hours, there remains 6 hours 15 minutes; therefore I ſearch by the Second Rule the Declination of the Sun for *Paris* on the 24th of *July*, at 6 a Clock 15 minutes after Noon, in the Year 1684, and I find 17 degrees, and about 13 minutes which is the Declination of the *Sun* for *Peking* on the day and hour propoſed.

Ano-

Another Example.

I would know at *Quebec* the Declination of the *Sun* on the 25*th* of *March* 1683, at 10 a Clock in the Morning, which being reduced, will be 22 Hours after Noon on the 24*th* of *March*, and I find in the Table that I must add for *Quebec* 4 Hours 36 minutes, therefore we have the 24*th* of *March* at 26 Hours 36 minutes, or the 25*th* of *March* at 2 a Clock 36 minutes after Noon. For which time in the Year 1683, I find the *Suns* Declination to be 5 degrees and about 30 minutes, which is that for *Quebec* on the day and hour required..

An

An *ADVERTISEMENT* *concerning the* Figures.

HEre you must obſerve that the *Dials* drawn in theſe Figures, are not made expreſly for ny place; for it is impoſſible to make the Magtude of the Lines which we muſt uſe equal to oſe that are here drawn: we ought only to folw the Precepts, and not to meaſure with the ompaſſes the length of the Lines, to ſee if ey agree: For *Example*, altho we ſay make e Line *aſ* equal to A S, yet theſe two Lines e not equal in the Figure, becauſe that ſomemes one of the ends of the Line A S as S being e Point of the Style which is not in the *Plane*, e apparent Magnitude of that Line is not the ue Magnitude, and it alſo happens often-times at the Lines of the Figure anſwers not among emſelves according to the Diſcourſe which was one to order the place of the Figures: it is ſufficent to obſerve well after what Manner, of what Mag-

Magnitude, and in what Angles we prefcrib
to draw the Lines without depending on th
Figure which ferves but to help the Imagina
tion, and to guide you in the Operations, feein
that it is almoft impoffible to meet with tw
like *Dials* among a great number which we mak
on the *Planes* propofed, as it is found ordinarily.

A TABLE Of the Suns Declination For the Year 1681,

Which was the First after Bissextile or Leap-year.

For the year 1681.

Days	January D.	M. South.	Diff. M
1	21	41	10
2	21	31	11
3	21	20	11
4	21	9	11
5	20	58	11
6	20	46	12
7	20	34	12
8	20	22	12
9	20	9	13
10	19	56	14
11	19	42	14
12	19	28	14
13	19	14	15
14	18	59	15
15	18	44	15
16	18	29	16
17	18	13	16
18	17	57	16
19	17	41	17
20	17	24	17
21	17	7	17
22	16	50	18
23	16	32	18
24	16	14	18
25	15	56	18
26	15	38	19
27	15	19	19
28	15	0	19
29	14	41	19
30	14	22	20
31	14	2	

Days	Februar. D.	M. South.	Diff. M
1	13	42	20
2	13	22	20
3	13	2	20
4	12	42	21
5	12	21	21
6	12	0	21
7	11	39	21
8	11	18	21
9	10	57	22
10	10	35	22
11	10	13	22
12	9	51	22
13	9	29	22
14	9	7	22
15	8	45	23
16	8	22	23
17	7	59	23
18	7	36	23
19	7	13	23
20	6	50	23
21	6	27	23
22	6	4	23
23	5	41	23
24	5	18	24
25	4	54	23
26	4	31	23
27	4	8	24
28	3	44	24

For the Year 1681.

Days	March D. So.	M. No.	Diff. M.	Days	April D. North	M.	Diff M.
1	3	20	23	1	8	39	22
2	2	57	24	2	9	1	22
3	2	33	23	3	9	23	21
4	2	10	23	4	9	44	21
5	1	47	24	5	10	5	21
6	1	23	24	6	10	26	21
7	0	59	24	7	10	47	21
8	0	35	24	8	11	8	21
9	0	11	23	9	11	29	21
10	0 No.	12	24	10	11	50	20
11	0	36	24	11	12	10	20
12	1	0	23	12	12	30	20
13	1	23	24	13	12	50	20
14	1	47	24	14	13	10	19
15	2	11	23	15	13	29	19
16	2	34	23	16	13	48	19
17	2	57	24	17	14	7	19
18	3	21	23	18	14	26	19
19	3	44	23	19	14	45	18
20	4	7	24	20	15	3	18
21	4	31	23	21	15	21	18
22	4	54	23	22	15	39	17
23	5	17	23	23	15	36	17
24	5	40	23	24	16	13	17
25	6	3	23	25	16	30	17
26	6	26	22	26	16	47	17
27	6	48	22	27	17	4	16
28	7	10	22	28	17	20	16
29	7	32	23	29	17	36	16
30	7	55	22	30	17	52	15
31	8	17					

For the Year 1681.

Days	May D. North	M.	Diff. M.	Days	June D. North	M.	Diff. M.
1	18	7	15	1	23	11	4
2	18	22	14	2	23	15	3
3	18	36	15	3	23	18	3
4	18	51	14	4	23	21	2
5	19	5	14	5	23	23	2
6	19	19	13	6	23	25	2
7	19	32	13	7	23	27	1
8	19	45	13	8	23	28	1
9	19	58	12	9	23	29	0
10	20	10	12	10	23	29	0
11	20	22	12	11	23	29	0
12	20	34	11	12	23	29	1
13	20	45	11	13	23	28	1
14	20	56	11	14	23	27	2
15	21	7	10	15	23	25	2
16	21	17	10	16	23	23	2
17	21	27	10	17	23	21	3
18	21	37	9	18	23	18	3
19	21	46	9	19	23	15	4
20	21	55	9	20	23	11	4
21	22	4	8	21	23	7	5
22	22	12	8	22	23	2	5
23	22	20	7	23	22	57	5
24	22	27	7	24	22	52	6
25	22	34	6	25	22	46	6
26	22	40	6	26	22	40	6
27	22	46	6	27	22	34	7
28	22	52	5	28	22	27	7
29	22	57	5	29	22	20	8
30	23	2	5	30	22	12	
31	23	7					

Days	July D. North	July M.	Diff. M.	Days	August D. North	August M.	Diff. M.
1	22	4	8	1	15	6	18
2	21	56	9	2	14	48	18
3	21	47	9	3	14	29	18
4	21	38	10	4	14	11	19
5	21	28	10	5	13	52	19
6	21	18	10	6	13	33	19
7	21	8	11	7	13	14	20
8	20	57	11	8	12	54	20
9	20	46	11	9	12	34	20
10	20	35	12	10	12	14	20
11	20	23	12	11	11	54	20
12	20	11	12	12	11	34	20
13	19	59	13	13	11	14	21
14	19	46	13	14	10	53	21
15	19	33	13	15	10	32	21
16	19	20	14	16	10	11	21
17	19	6	14	17	9	50	21
18	18	52	14	18	9	29	21
19	18	38	15	19	9	8	22
20	18	23	15	20	8	46	22
21	18	8	15	21	8	24	22
22	17	53	15	22	8	2	22
23	17	38	16	23	7	40	22
24	17	22	16	24	7	18	22
25	17	6	16	25	6	56	22
26	16	50	17	26	6	34	23
27	16	33	17	27	6	11	23
28	16	16	17	28	6	48	23
29	15	59	17	29	5	25	23
30	15	42	18	30	5	2	23
31	15	24		31	4	39	

Days	September D. No.	September M. So.	Diff. M.	Days	October D. South	October M. South	Diff. M.
1	4	16	23	1	7	22	22
2	3	53	23	2	7	44	23
3	3	30	23	3	8	7	22
4	3	7	23	4	8	29	22
5	2	44	23	5	8	51	22
6	2	21	23	6	9	13	22
7	1	58	24	7	9	35	22
8	1	34	23	8	9	57	22
9	1	11	24	9	10	19	22
10	0	47	23	10	10	41	21
11	0	24	24	11	11	2	21
12	0 South	0	23	12	11	23	21
13	0	23	24	13	11	44	21
14	0	47	23	14	12	5	21
15	1	10	24	15	12	26	21
16	1	34	23	16	12	47	20
17	1	57	24	17	13	7	20
18	2	21	23	18	13	27	20
19	2	44	23	19	13	47	20
20	3	7	24	20	14	7	19
21	3	31	23	21	14	26	19
22	3	54	23	22	14	45	19
23	4	17	24	23	15	4	19
24	4	41	23	24	15	23	19
25	5	4	23	25	15	42	18
26	5	27	23	26	16	0	18
27	5	50	23	27	16	18	18
28	6	13	23	28	16	36	17
29	6	36	23	29	16	53	17
30	6	59	23	30	17	10	17
				31	17	27	16

For the Year 1681.

Days	November D. South	M.	Diff. M	Days	December M. South	D.	Diff. M.
1	17	43	16	1	23	8	4
2	17	59	16	2	23	12	4
3	18	15	16	3	23	16	3
4	18	31	15	4	23	19	3
5	18	46	15	5	23	22	2
6	19	1	15	6	23	24	2
7	19	16	14	7	23	26	2
8	19	30	14	8	23	28	1
9	19	44	13	9	23	29	0
10	19	57	13	10	23	29	0
11	20	10	13	11	23	29	0
12	20	23	12	12	23	29	1
13	20	35	12	13	23	28	1
14	20	47	12	14	23	27	2
15	20	59	11	15	23	25	2
16	21	10	11	16	23	23	3
17	21	21	10	17	23	20	3
18	21	31	10	18	23	17	4
19	21	41	10	19	23	13	4
20	21	51	9	20	23	9	5
21	22	0	9	21	23	4	5
22	22	9	8	22	22	59	6
23	22	17	8	23	22	53	6
24	22	25	7	24	22	47	6
25	22	32	7	25	22	41	7
26	22	39	7	26	22	34	7
27	22	46	6	27	22	27	8
28	22	52	6	28	22	19	8
29	22	58	5	29	22	11	9
30	23	3	5	30	22	2	9
				31	21	53	10

A
TABLE
Of the Suns Declination
For the Year 1682.
Being the Second after
Biſſextile or Leap-Year.

For the Year 1682.

Days	January D.	M. South	Diff. M.	Days	February D.	M. South	Diff. M.
1	21	43	10	1	13	47	20
2	21	33	10	2	13	27	20
3	21	23	11	4	13	7	21
4	21	12	11	4	12	46	20
5	21	1	12	5	12	26	21
6	20	49	12	6	12	5	21
7	20	37	12	7	11	44	21
8	20	25	13	8	11	23	22
9	20	12	13	9	11	1	21
10	19	59	14	10	10	40	22
11	19	45	14	11	10	18	22
12	19	31	14	12	9	56	22
13	19	17	14	13	9	34	22
14	19	3	15	14	9	12	22
15	18	48	15	15	8	50	23
16	18	33	16	16	8	27	22
17	18	17	16	17	8	5	23
18	18	1	16	18	7	42	23
19	17	45	17	19	7	19	23
20	17	28	17	20	6	56	23
21	17	11	17	21	6	33	23
22	16	54	17	22	6	10	23
23	16	37	18	23	5	47	23
24	16	19	18	24	5	24	24
25	16	1	18	25	5	0	23
26	15	43	19	26	4	37	24
27	15	24	19	27	4	13	23
28	15	5	19	28	3	50	24
29	14	46	19				
30	14	27	20				
31	14	7					

For the Year 1682.

Days	March D.	M.	Diff. M.	Days	April D.	M.	Diff. M.
	So.	No.			North		
1	3	26	23	1	8	33	22
2	3	3	24	2	8	55	22
3	2	39	24	3	9	17	22
4	2	15	23	4	9	39	21
5	1	52	24	5	10	0	21
6	1	28	24	6	10	21	21
7	1	4	23	7	10	42	21
8	0	41	24	8	11	3	21
9	0	17	24	9	11	24	21
10	0 No.	7	23	10	11	45	20
11	0	30	24	11	12	5	20
12	0	54	24	12	12	25	20
13	1	18	23	13	12	45	20
14	1	41	24	14	13	5	19
15	2	5	23	15	13	24	19
16	2	28	24	16	13	43	19
17	2	52	23	17	14	2	19
18	3	15	23	18	14	21	19
19	3	38	24	19	14	40	18
20	4	2	23	20	14	58	18
21	4	25	23	21	15	16	18
22	4	48	23	22	15	34	18
23	5	11	23	23	15	52	17
24	5	34	23	24	16	9	17
25	5	57	23	25	16	26	17
26	6	20	22	26	16	43	17
27	6	42	23	27	17	0	16
28	7	5	22	28	17	16	16
29	7	27	22	29	17	32	16
30	7	49	22	30	17	48	15
31	8	11	22				

Days	May D. North	May M.	Diff. M.	Days	June D. North	June M.	Diff. M.
1	18	3	15	1	23	10	4
2	18	18	15	2	23	14	3
3	18	33	14	3	23	17	3
4	18	47	14	4	23	20	3
5	19	1	14	5	23	23	2
6	19	15	14	6	23	25	2
7	19	29	13	7	23	27	1
8	19	42	13	8	23	28	1
9	19	55	12	9	23	29	0
10	20	7	12	10	23	29	0
11	20	19	12	11	23	29	0
12	20	31	12	12	23	29	1
13	20	43	11	13	23	28	1
14	20	54	11	14	23	27	1
15	21	5	10	15	23	26	2
16	21	15	10	16	23	14	2
17	21	25	10	17	23	22	3
18	21	35	9	18	23	19	3
19	21	44	9	19	23	16	4
20	21	53	9	20	23	12	4
21	21	2	8	21	23	8	4
22	22	10	8	22	23	4	5
23	22	18	7	23	22	59	5
24	22	25	7	24	22	54	6
25	22	32	7	25	22	48	6
26	22	39	6	26	22	42	6
27	22	45	6	27	22	36	7
28	22	51	5	28	22	29	7
29	22	56	5	29	22	22	8
30	23	1	5	30	22	14	8
31	23	6					

Days	*July* D.	M.	Diff. M.
	North.		
1	22	6	
2	21	58	8
3	21	49	9
4	21	40	9
5	21	30	10
6	21	20	10
7	21	10	10
8	21	0	10
9	20	49	11
10	20	38	11
11	20	26	12
12	20	14	12
13	20	2	12
14	19	49	13
15	19	36	13
16	19	23	13
17	19	9	14
18	18	55	14
19	18	41	14
20	18	27	14
21	18	12	15
22	17	57	15
23	17	41	16
24	17	25	16
25	17	9	16
26	16	53	16
27	16	37	16
28	16	20	17
29	16	3	17
30	15	46	17
31	15	28	18

Days	*August* D.	M.	Diff. M.
	North.		
1	15	10	
2	14	52	18
3	14	34	18
4	14	15	19
5	13	56	19
6	13	37	19
7	13	18	19
8	12	59	19
9	12	39	20
10	12	19	20
11	11	59	20
12	11	39	20
13	11	19	20
14	10	58	21
15	10	37	21
16	10	16	21
17	9	55	21
18	9	34	21
19	9	12	21
20	8	51	22
21	8	29	22
22	8	7	22
23	7	45	22
24	7	23	22
25	7	1	22
26	6	38	23
27	6	16	22
28	5	54	22
29	5	31	23
30	5	8	23
31	4	45	23

Days	September D. No.	M. So	Diff. M.
1	4	22	23
2	3	59	23
3	3	36	23
4	3	13	23
5	2	50	24
6	2	26	23
7	2	3	23
8	1	40	24
9	1	16	23
10	0	53	23
11	0	30	24
12	0 South	6	23
13	0	17	24
14	0	41	23
15	1	4	24
16	1	28	24
17	1	52	23
18	2	15	23
19	2	38	23
20	3	2	23
21	3	25	23
22	3	48	24
23	4	12	23
24	4	35	23
25	4	58	24
26	5	22	23
27	5	45	23
28	6	8	23
29	6	31	22
30	6	53	23

Days	October D. South	M.	Diff. M.
1	7	16	23
2	7	39	22
3	8	1	23
4	8	24	22
5	8	46	22
6	9	8	22
7	9	30	22
8	9	52	22
9	10	14	22
10	10	36	21
11	10	57	21
12	11	18	21
13	11	39	21
14	12	0	21
15	12	21	21
16	12	42	20
17	13	2	20
18	13	22	20
19	13	42	20
20	14	2	20
21	14	22	19
22	14	41	19
23	15	0	19
24	15	19	18
25	15	37	18
26	15	55	18
27	16	13	18
28	16	31	18
29	16	49	17
30	17	6	17
31	17	23	

For the Year 1682.

Days	November D. South	M.	Diff. M.	Days	December M. South	D.	Diff. M.
1	17	40	16	1	23	7	4
2	17	56	16	2	23	11	4
3	18	12	15	3	23	15	3
4	18	27	15	4	23	18	3
5	18	42	15	5	23	21	3
6	18	57	15	6	23	24	2
7	19	12	14	7	23	26	2
8	19	26	14	8	23	28	1
9	19	40	14	9	23	29	0
10	19	54	13	10	23	29	0
11	20	7	13	11	23	29	0
12	20	20	12	12	23	29	1
13	20	32	12	13	23	28	1
14	20	44	12	14	23	27	2
15	20	56	11	15	23	25	2
16	21	7	11	16	23	23	3
17	21	18	11	17	23	20	3
18	21	29	10	18	23	17	3
19	21	39	10	19	23	14	4
20	21	49	9	20	23	10	5
21	21	58	9	21	23	5	5
22	22	7	8	22	23	0	5
23	22	15	8	23	22	55	6
24	22	23	8	24	22	49	7
25	22	31	7	25	22	42	7
26	22	38	7	26	22	35	7
27	22	45	6	27	22	28	8
28	22	51	6	28	22	20	8
29	22	57	5	29	22	12	8
30	23	2	5	30	22	4	9
				31	21	55	

A
TABLE
Of the Suns Declination
For the Year 1683.
Being the Third after
Bissextile or Leap-Year.

For the Year 1683.

Days	January D.	M.	Diff. M	Days	February D.	M.	Diff. M
	South				South		
1	21	46	10	1	13	52	20
2	21	36	10	2	13	32	20
3	21	26	11	4	13	12	20
4	21	15	11	4	12	52	21
5	21	4	12	5	12	31	21
6	20	52	12	6	12	10	21
7	20	40	12	7	11	49	21
8	20	28	13	8	11	28	21
9	20	15	13	9	11	7	22
10	20	2	13	10	10	45	22
11	19	49	14	11	10	23	22
12	19	35	14	12	10	11	22
13	19	21	15	13	9	39	22
14	19	6	15	14	9	17	22
15	18	51	15	15	8	55	22
16	18	36	15	16	8	33	23
17	18	21	16	17	8	10	23
18	18	5	16	18	7	47	23
19	17	49	17	19	7	24	23
20	17	32	17	20	7	1	23
21	17	15	17	21	6	38	23
22	16	58	17	22	6	15	23
23	16	51	18	23	5	52	23
24	16	23	18	24	5	29	23
25	16	5	18	25	5	6	24
26	15	47	18	26	4	42	23
27	15	29	19	27	4	19	23
28	15	10	19	28	3	56	24
29	14	51	19				
30	14	32	20				
31	14	12					

For the Year 1683.

Days	March D. So.	M. No.	Diff M.
1	3	32	24
2	3	8	23
3	2	45	23
4	2	21	24
5	1	57	24
6	1	34	23
7	1	10	24
8	0	46	23
9	0	23	24
10	0	No. 1	24
11	0	25	24
12	0	49	23
13	1	12	24
14	1	36	23
15	1	59	24
16	2	23	23
17	2	46	24
18	3	10	23
19	3	33	23
20	3	56	23
21	4	19	23
22	4	42	23
23	5	5	23
24	5	28	23
25	5	51	23
26	6	14	22
27	6	37	23
28	6	59	23
29	7	22	22
30	7	44	22
31	8	6	22

Days	April D. North	M.	Diff M.
1	8	28	22
2	8	50	22
3	9	12	22
4	9	34	21
5	9	55	21
6	10	16	21
7	10	37	21
8	10	58	21
9	11	19	21
10	11	40	20
11	12	0	20
12	12	20	20
13	12	40	20
14	13	0	20
15	13	20	19
16	13	39	19
17	13	58	19
18	14	17	19
19	14	36	18
20	14	54	18
21	15	12	18
22	15	30	18
23	15	48	17
24	16	5	17
25	16	22	17
26	16	39	17
27	16	56	16
28	17	12	16
29	17	28	16
30	17	44	15

For the Year 1683.

Days	May D.	May M. North	Diff. M.	Days	June D.	June M. North	Diff. M.
1	17	59	15	1	23	9	4
2	18	14	15	2	23	13	3
3	18	29	14	3	23	16	3
4	18	44	14	4	23	19	3
5	18	58	14	5	23	22	2
6	19	12	14	6	23	24	2
7	19	26	13	7	23	26	1
8	19	39	13	8	23	27	1
9	19	52	12	9	23	28	1
10	19	5	12	10	23	29	0
11	20	17	12	11	23	29	0
12	20	29	12	12	23	29	1
13	20	40	11	13	23	28	1
14	20	51	11	14	23	27	1
15	20	2	10	15	23	26	2
16	21	13	10	16	23	24	2
17	21	23	10	17	23	22	3
18	21	33	9	18	23	19	3
19	21	42	9	19	23	16	3
20	21	51	9	20	23	13	4
21	22	0	8	21	23	9	4
22	22	8	8	22	23	5	5
23	22	16	7	23	23	0	5
24	22	23	7	24	22	55	6
25	22	30	7	25	22	49	6
26	22	37	6	26	22	43	6
27	22	43	6	27	22	37	7
28	22	49	6	28	22	30	7
29	22	55	5	29	22	23	7
30	23	0	5	30	22	16	8
31	23	5	4				

Days	July D. North.	M.	Diff. M.	Days	August D. North.	M	Diff. M
1	22	8	8	1	15	14	18
2	22	0	9	2	14	56	18
3	21	51	9	3	14	38	18
4	21	42	9	4	14	20	19
5	21	33	10	5	14	1	19
6	21	23	10	6	13	42	19
7	21	13	11	7	13	23	19
8	21	2	11	8	13	4	20
9	21	51	11	9	12	44	20
10	20	40	11	10	12	24	20
11	20	29	12	11	12	4	20
12	20	17	12	12	11	44	20
13	20	5	13	13	11	24	21
14	19	52	13	14	11	3	21
15	19	39	13	15	10	42	21
16	19	26	13	16	10	21	21
17	19	13	14	17	10	0	21
18	18	59	14	18	9	39	21
19	18	45	15	19	9	18	22
20	18	30	15	20	8	56	22
21	18	15	15	21	8	34	22
22	18	0	15	22	8	12	22
23	17	45	16	23	7	50	22
24	17	29	16	24	7	28	22
25	17	13	16	25	7	6	22
26	16	57	16	26	6	44	22
27	16	41	17	27	6	22	23
28	16	24	17	28	5	59	23
29	16	7	17	29	5	36	23
30	15	50	18	30	5	13	22
31	15	32		31	4	51	23

Days	September D. No.	September M. So.	Diff. M.
1	4	28	23
2	4	5	23
3	3	42	23
4	3	19	24
5	2	55	23
6	2	32	23
7	2	9	24
8	1	45	23
9	1	22	23
10	0	59	24
11	0	35	23
12	0 South	12	24
13	0	12	23
14	0	35	24
15	0	59	23
16	1	22	24
17	1	46	23
18	2	9	24
19	2	33	23
20	2	56	23
21	3	19	24
22	3	43	23
23	4	6	23
24	4	29	24
25	4	53	23
26	5	16	23
27	5	39	23
28	6	2	23
29	6	25	23
30	6	48	23

Days	October South D.	October South M.	Diff. M
1	7	11	22
2	7	33	23
3	7	56	22
4	8	18	23
5	8	41	22
6	9	3	22
7	9	25	22
8	9	47	22
9	10	9	21
10	10	30	22
11	10	52	21
12	11	13	21
13	11	34	21
14	11	55	21
15	12	16	21
16	12	37	20
17	12	57	20
18	13	17	20
19	13	37	20
20	13	57	20
21	14	7	19
22	14	36	19
23	14	55	19
24	15	14	19
25	15	33	18
26	15	51	18
27	16	9	18
28	16	27	17
29	16	44	17
30	17	1	17
31	17	18	17

Days	November D. South	M.	Diff. M.
1	17	35	16
2	17	51	16
3	18	7	16
4	18	23	16
5	18	39	15
6	18	54	15
7	19	9	14
8	19	23	14
9	19	37	14
10	19	51	13
11	20	4	13
12	20	17	13
13	20	30	12
14	20	42	12
15	20	54	11
16	21	5	11
17	21	16	11
18	21	27	10
19	21	37	10
20	21	47	9
21	21	56	9
22	22	5	8
23	22	13	8
24	22	21	8
25	22	29	7
26	22	36	7
27	22	43	6
28	22	49	6
29	22	55	6
30	23	1	5

Days	December M. South	D.	Diff M.
1	23	6	4
2	23	10	4
3	23	14	4
4	23	18	3
5	23	21	2
6	23	23	2
7	23	25	2
8	23	27	1
9	23	28	1
10	23	29	0
11	23	29	0
12	23	29	1
13	23	28	1
14	23	27	1
15	23	26	2
16	23	24	3
17	23	21	3
18	23	18	3
19	23	15	4
20	23	11	5
21	23	6	5
22	23	1	5
23	22	56	6
24	22	50	6
25	22	44	7
26	22	37	7
27	22	30	8
28	22	22	8
29	22	14	8
30	22	6	9
31	21	57	

A TABLE

Of the Suns Declination

For the Year 1684.

Being

Biſſextile or Leap-Year.

Days	January D.	M. South	Diff. M.
1	21	48	10
2	21	38	10
3	21	28	11
4	21	17	11
5	21	6	11
6	20	55	12
7	20	43	12
8	20	31	13
9	20	18	13
10	20	5	13
11	19	52	13
12	19	39	14
13	19	25	15
14	19	10	15
15	18	55	15
16	18	40	15
17	18	25	16
18	18	9	16
19	17	53	16
20	17	37	17
21	17	20	17
22	17	3	17
23	16	46	18
24	16	28	18
25	16	10	18
26	15	52	19
27	15	33	19
28	15	14	19
29	14	55	19
30	14	36	19
31	14	17	

Days	February D.	M. South	Diff. M.
1	13	57	20
2	13	37	20
4	13	17	20
4	12	57	21
5	12	36	21
6	12	15	21
7	11	54	21
8	11	33	21
9	11	12	21
10	10	51	22
11	10	29	22
12	10	7	22
13	9	45	22
14	9	23	22
15	9	1	23
16	8	38	22
17	8	16	23
18	7	53	23
19	7	30	23
20	7	7	23
21	6	44	23
22	6	21	23
23	5	58	23
24	5	35	23
25	5	12	23
26	4	49	24
27	4	25	23
28	4	2	24
	3	38	23

Days	March D. So.	March M. No.	Diff. M.
1	3	15	24
2	2	51	24
3	2	27	23
4	2	4	24
5	1	40	24
6	1	16	24
7	0	52	23
8	0	29	24
9	0	5	24
10	0 No.	19	23
11	0	42	24
12	1	6	23
13	1	29	24
14	1	53	24
15	2	17	23
16	2	40	24
17	3	4	23
18	3	27	23
19	3	50	23
20	4	13	24
21	4	37	23
22	5	0	23
23	5	23	23
24	5	45	22
25	6	8	23
26	6	31	22
27	6	53	23
28	7	16	22
29	7	38	22
30	8	0	22
31	8	22	

Days	April D. North	April M.	Diff. M.
1	8	44	22
2	9	6	22
3	9	28	21
4	9	49	22
5	10	11	21
6	10	32	21
7	10	53	21
8	11	14	20
9	11	34	21
10	11	55	20
11	12	15	20
12	12	35	20
13	12	55	20
14	13	15	19
15	13	34	19
16	13	53	19
17	14	12	19
18	14	31	18
19	14	49	18
20	15	7	18
21	15	25	18
22	15	43	18
23	16	1	17
24	16	18	17
25	16	35	17
26	16	52	16
27	17	8	16
28	17	24	16
29	17	40	15
30	17	55	15

Days	May D. North	May M. North	Diff. M.	Days	June D. North	June M. North	Diff. M.
1	18	10	15	1	23	12	4
2	18	25	15	2	23	16	3
3	18	40	14	3	23	19	3
4	18	54	14	4	23	21	3
5	19	8	14	5	23	24	2
6	19	22	13	6	23	26	1
7	19	35	13	7	23	27	1
8	19	48	13	8	23	28	1
9	20	1	12	9	23	29	0
10	20	13	12	10	23	29	0
11	20	25	12	11	23	29	0
12	20	37	11	12	23	29	1
13	20	48	11	13	23	28	1
14	20	59	11	14	23	27	2
15	21	10	10	15	23	25	2
16	21	20	10	16	23	23	3
17	21	30	10	17	23	20	3
18	21	40	9	18	23	17	3
19	21	49	9	19	23	14	4
20	21	58	8	20	23	10	4
21	22	6	8	21	23	6	5
22	22	14	7	22	23	1	5
23	22	21	7	23	22	56	5
24	22	28	7	24	22	51	6
25	22	35	7	25	22	45	6
26	22	42	6	26	22	39	7
27	22	48	5	27	22	32	7
28	22	53	5	28	22	25	7
29	22	58	5	29	22	18	8
30	23	3	5	30	22	10	8
31	23	8	4				

Days	July D. North	July M. North	Diff. M.	Day	August D. North	August M. North	Diff. M.
1	22	2	9	1	15	1	18
2	21	53	9	2	14	43	18
3	21	44	9	3	14	25	19
4	21	35	9	4	14	6	19
5	21	26	10	5	13	47	19
6	21	16	11	6	13	28	19
7	21	5	11	7	13	9	20
8	20	54	11	8	12	49	20
9	20	43	11	9	12	29	20
10	20	32	12	10	12	9	20
11	20	20	12	11	11	49	20
12	20	8	12	12	11	29	21
13	19	56	13	13	11	8	20
14	19	43	13	14	10	48	21
15	19	30	14	15	10	27	21
16	19	16	14	16	10	6	21
17	19	2	14	17	9	45	22
18	18	48	14	18	9	23	21
19	18	34	15	19	9	2	22
20	18	19	15	20	8	40	22
21	18	4	15	21	8	18	22
22	17	49	16	22	7	56	22
23	17	33	16	23	7	34	22
24	17	17	16	24	7	12	22
25	17	1	16	25	6	50	23
26	16	45	17	26	6	27	22
27	16	28	17	27	6	5	23
28	16	11	17	28	5	42	23
29	15	54	17	29	5	19	22
30	15	37	18	30	4	57	23
31	15	19		31	4	34	

For the Year 1684.

Days	September D. No.	M. So.	Diff. M.
1	4	11	23
2	4	48	23
3	3	25	24
4	3	1	23
5	2	38	23
6	2	15	23
7	1	52	24
8	1	28	23
9	1	5	24
10	0	41	23
11	0	18	24
12	0 South	6	23
13	0	29	24
14	0	53	23
15	1	16	24
16	1	40	23
17	2	3	24
18	2	27	23
19	2	50	24
20	3	14	23
21	3	37	23
22	4	0	24
23	4	24	23
24	4	47	23
25	5	10	23
26	5	33	23
27	5	56	23
28	6	19	23
29	6	42	23
30	7	5	23

Days	October D. South	M.	Diff. M.
1	7	28	22
2	7	50	23
3	8	13	22
4	8	35	22
5	8	57	22
6	9	19	22
7	9	41	22
8	10	3	22
9	10	25	21
10	10	46	22
11	11	8	21
12	11	29	21
13	11	50	21
14	12	11	21
15	12	32	20
16	12	52	20
17	13	12	20
18	13	32	20
19	13	52	20
20	14	12	19
21	14	31	19
22	14	50	19
23	15	9	19
24	15	28	18
25	15	46	18
26	16	4	18
27	16	22	18
28	16	40	17
29	16	57	17
30	17	14	17
31	17	31	

For the Year 1684.

Days	November D. South	M.	Diff. M.	Days	December M. South	D.	Diff M.
1	17	47	16	1	23	9	4
2	18	3	16	2	23	13	4
3	18	19	16	3	23	17	3
4	18	35	15	4	23	20	3
5	18	50	15	5	23	23	2
6	19	5	14	6	23	25	2
7	19	19	14	7	23	27	1
8	19	33	14	8	23	28	1
9	19	47	14	9	23	29	0
10	20	1	13	10	23	29	0
11	20	14	12	11	23	29	0
12	20	26	13	12	23	29	1
13	20	39	12	13	23	28	2
14	20	51	11	14	23	26	2
15	21	2	11	15	23	24	2
16	21	13	11	16	23	22	3
17	21	24	10	17	23	19	3
18	21	34	10	18	23	16	4
19	21	44	10	19	23	12	4
20	21	54	9	20	23	8	5
21	22	3	8	21	23	3	5
22	22	11	8	22	22	57	5
23	22	19	8	23	22	52	6
24	22	27	7	24	22	46	7
25	22	34	7	25	22	39	7
26	22	41	7	26	22	32	8
27	22	48	6	27	22	24	8
28	22	54	5	28	22	16	8
29	22	59	5	29	22	8	9
30	23	4		30	21	59	9
				31	21	50	

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TABLE
Of the Differences of Meridians
Of the most considerable Places
In the whole World,
In respect of *PARIS*,
With the Height of the Pole or Latitude of the same places.

Names of places.	*Differences of Meridians* H.	M.		*Height of the Po[le] or Latitude* D.	M.	
Abbeville	0	1	add	50	0	Nor[th]
Agra	5	26	ſubſt.	26	0	
Aiz Prov.	0	14		43	33	
Aleppo	2	50		36	46	
Alexandria Æ.	2	13		30	58	
Amiens	0	0		49	55	
Amſterdam	0	14		52	21	
Angers	0	11	add	47	32	
Antwerp	0	15	ſubſt.	51	12	
Arles	0	11		43	36	
Arras	0	2		50	9	
Archangello	2	49		65	30	
Athens	1	35		7	40	
Avignon	0	11		43	52	
Bayonne	0	15	add	43	30	
Blois	0	3		47	46	
Bordeaux	0	10		44	50	
Bourges	0	0		47	20	
Boulogne It.	0	40	ſubſt.	44	20	
Breſt	0	28	add	48	23	
Bruſſels	0	15	ſubſt.	50	48	

Names of Places	*Differences of Meridians* H.	M.		*Height of the Pole or Latitude* D.	M.	
Cadis	0	37	add	36	16	
Caen	0	9		49	11	
Callais	0	2		50	57	
Camboge	6	53	ſubſt.	11	20	
Cambray	0	3		50	5	
Cap. of b. Eſ.	1	11		34	32	Soutth
Cap-Verd	1	34	add	14	27	North
Chartres	0	4		48	30	
Conſtantinop.	2	0	ſubſt.	42	56	
Copenhagen	0	42		55	43	
Cracovi	1	14		50	10	
Dantzick	1	12	ſubſt.	54	22	
Dieppe	0	4	add	49	56	
Dijon	0	12	ſubſt.	47	28	
Dole	0	13		47	20	
Doway	0	3		50	15	
Dunkerk	0	0		51	1½	
Ferare	0	41	ſubſt.	44	54	
Ile de Fer.	1	38	add	28	10	
Fez	0	31		33	10	
La Fle che	0	10		47	42	

The Differences of Meridians.

Names of Places.	Differences of Meridians H.	M.		Height of the Pole or Latitude D.	M.	
Florence	o	40½	ſubſt.	43	41	
Francfort M.	o	27		50	4	
Fribourg	o	25		48	16	
Gaunt	o	6		51	2	
Geneva	o	17		46	20	
Goa	4	58		15	30	
Grenoble	o	15		45	11	
Hambourgh	o	34		53	43	
Havre de Gr.	o	8	add	49	7	
Hieruſalem	2	36	ſubſt.	32	o	
Hiſpaam	4	16		36	14	
Rochelle	o	13	add	46	11	
Leiden	o	12	ſubſt.	52	12	
Lima Per.	5	31	add	12	20	South
Lisbon	o	50		38	40	North
London	o	8		51	32	
Loudun	o	8		48	o	
Lovain	o	13	ſubſt.	50	50	
Lucques	o	37		43	40	
Luxembourg	o	20		49	38	
Lyons	o	11		45	46	

Names of Places.	Differences of Meridians D.	M.		Height of the Pole or Latitude D.	M.
Macon	0	11	ſubſt.	46	20
Madrid	0	25	add	40	10
Malta	0	53	ſubſt.	35	40
Malaca	6	45		2	20
S. Malo	0	18	add	48	39
Mantove	0	37	ſubſt.	45	1
Marſeilles	0	14		43	15
Martinique	4	15	add	14	14
Meaux	0	2	ſubſt.	48	56
Meſſina	0	58		38	21
Metz	0	19		49	10
Meſſique	7	8	add	20	30
Millan	0	31	ſubſt.	45	15
Modene	0	38		44	39
Monaco	0	23		43	39
Montpellier	0	6		43	37
Munick	0	50		48	58
Namur	0	13	ſubſt.	50	26
Nancy	0	19		48	39
Nantes	0	16		46	13
Naples	0	56		41	5
Narbonne	0	3		43	6
Nevers	0	2		47	19

Names of Places.	*Differences of Meridians.* H.	M.		*Height of the Pole or Latitude* D.	M.
Ostend	0	3	subst.	51	16
Orleans	0	2	add	47	54
Ormus	4	0	subst.	27	35
Paris	0	0		48	51
Parma	0	36	subst.	44	45
Pavia	0	31		44	58
Padoua	0	42		45	31
Pexing Chi.	7	45		40	0
Perigeux	0	9	add	45	34
Perpignan	0	5	subst.	45	52
Pernanibouc.	2	57	add	7	40 South
Perouge	0	44	subst.	42	56 North
Pisa	0	38		43	9
Plaisance	0	33		44	53
Poictiers	0	6	add	47	7
Porto-belo.	5	41		9	55
Prague	0	58½	subst.	50	40½
Quebec	4	36	add	47	0
S. Quentin.	0	6	subst.	49	46
Rennes	0	16	add	47	58
Rhemes	0	9	subst.	49	12
Riga	1	31		56	52

The Differences of Meridians.

Names of Places.	Differences of Meridians H.	M.		Height of the Pole or Latitude D.	M.
Rome	o	47	ſubſt.	41	54
Rouen	o	4	add	49	$27\frac{1}{2}$
Saintes	o	$11\frac{1}{2}$	add	45	38
Savone	o	28	ſubſt.	44	18
Sienne	o	41		43	11
Siras	3	42		34	14
Smirna	2	56		38	22
Strasbourgh	o	24		48	31
Stettin	o	54		53	34
Stockholm	1	7		59	30
Tangier	o	56	add	35	25
Tholouſe	o	2		43	29
Toleda	o	26		39	52
Tours	o	6		47	35
Tournay	o	5	ſubſt.	50	32
Tutin	o	25		44	9
Toulon	o	17		43	
Valencienne	o	4	ſubſt.	50	20
Verdun	o	38		45	33
Vienna	1	2		48	22

ADVERTISEMENT.

I Have begun to travel for the Impreſſion of an intire Work of the *Conique Sections*, where you ſhall find not only all the moſt excellent things has been diſcovered in this patt of the *Mathematicks*, but alſo a great number of new Properties which I have diſcovered, and whereof I have publiſhed every one upon different occaſions.

The Method of the Principal Demonſtration is particular to my ſelf, and I have uſed it to abridge very much this whole Work; I have given an Eſſay of this New Method, which I cauſed to be Printed, concerning the *Conique Sections* in the Year 1675.

See here in few words the order of theſe Books and what they contain.

The Firſt Book contains the *Lemma's* that are neceſſary for this Method.

The Second contains the original of the *Three Conique Sections*, with the Properties of their Diameters, and all that which depends on a Line cut harmonically or into two equal parts, the original of the *Aſymptotes*, and in the end divers Problems depending on theſe Principles.

The

The Third explains the relation of the *Ordinate Lines* to their Diameters, with the Rect-angles of the parts of the Diameters, Parameters, and that which concerns the Tangents.

The Fourth makes that appear which is moſt conſiderable touching the *Aſymptotes.*

The Fifth Book is fill'd with near 40 moſt curious Propoſitions upon the *Conique Sections.*

In the Sixth Book is treated of equal and like *Sections:*

The Seventh is continued all along on that which is called the leaſt and greateſt.

The Eighth treats of the *Foci* of the *Three Sections.*

In the Ninth Book is taught certain, moſt plain, and moſt uſeful Methods for the Deſcription of theſe *Sections.*

Then I ſhew you in an Appendix after what manner we ought to reſolve the *Conique Sections* that have for their *Baſes*, *Paraboles*, *Hyperboles* and *Elipſes*, and alſo all *Cylinders* of the ſame *Species.*

Afterwards I demonſtrate that by the ſame Method, and without difficulty, we may demonſtrate all the Properties of the *Conique Sections* of whatſoever compounded kind they are, I draw them from their *Cones* or *Pyramids*, which I diſtinguiſhed by Cones of the firſt kind, of the ſecond kind, of the third kind, *&c.* Theſe ſorts of *Cones* have for their *Baſes Circles* of all theſe kinds, and then having

ving explained after what manner we may render all the *Lemma's* of the firſt Book univerſal, I explain in the ſame manner that which is the *Second* Book, and in one part of the following the Sections of all *Cones* of any kind.

The Demonſtrations are the ſame as for the *Sections* of the firſt *Cone*, which is that which has the Circle of the firſt kind for its baſe: After that I ſee not any thing that we can deſire more univerſal, nor more eaſie on this ſame *Atiere*.

The

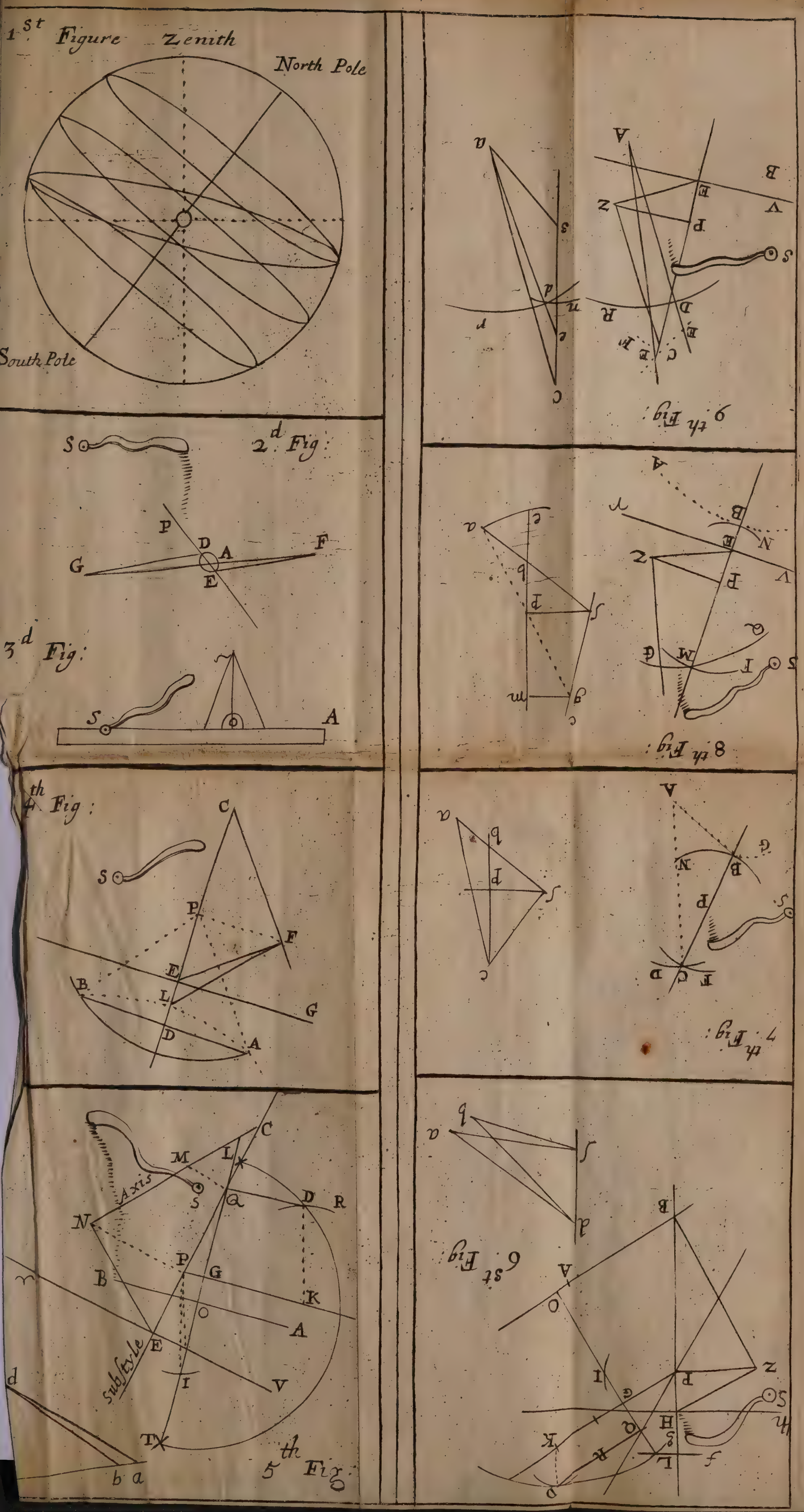
The Approbation.

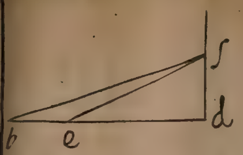
THis Treatise of the Gnomiques, or the Art of Drawing of Sun Dials, composed by Mr. De la Hire, *hath been read to the Assembly of the* Academy Royal of Sciences, *made the 9th of* May 1682.

J. B. Du Hamel *Secretary of the* Academy *of* Sciences.

THe Approbation seen, permitted to be Imprinted, made the 23th of *May* 1682.

DE LA REYNIE.


1st Figure
Zenith
North Pole
South Pole
2d Fig:
3d Fig:
4th Fig:
Axis
substyle
5th Fig
6st Fig:
7th Fig:
8th Fig:
9th Fig:



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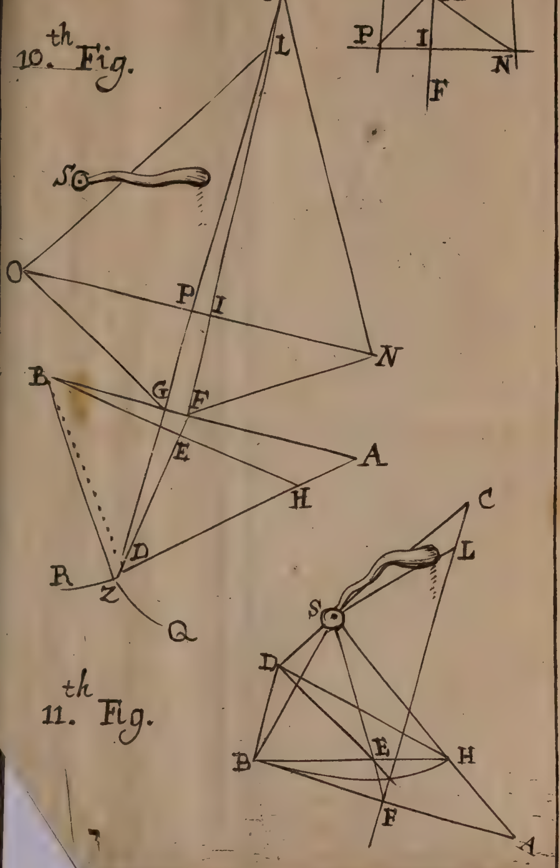

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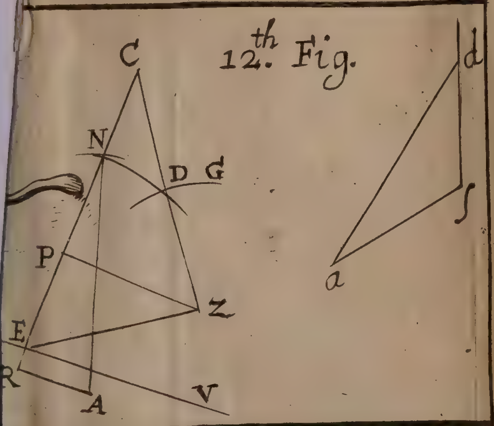

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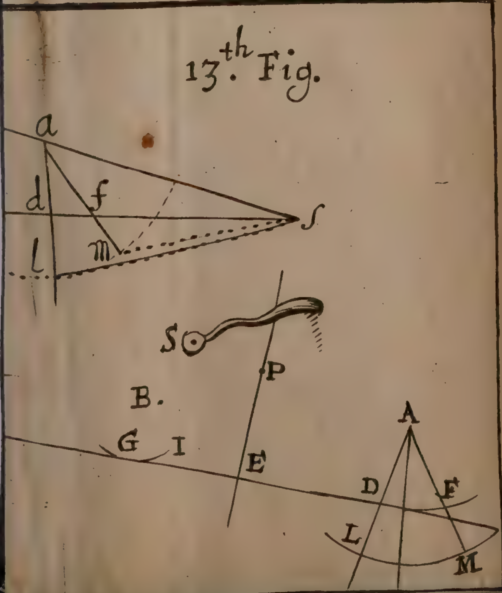

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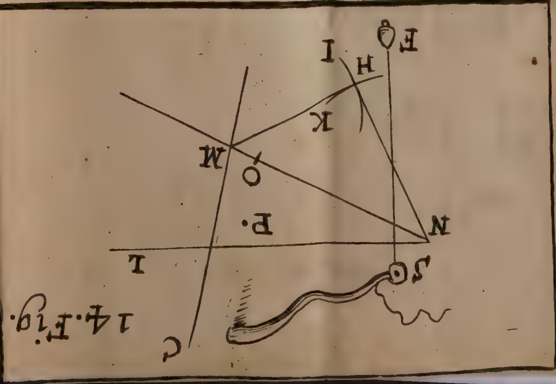

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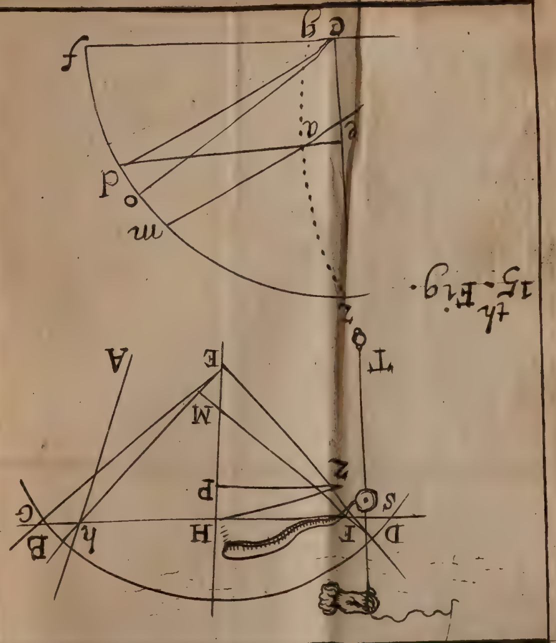

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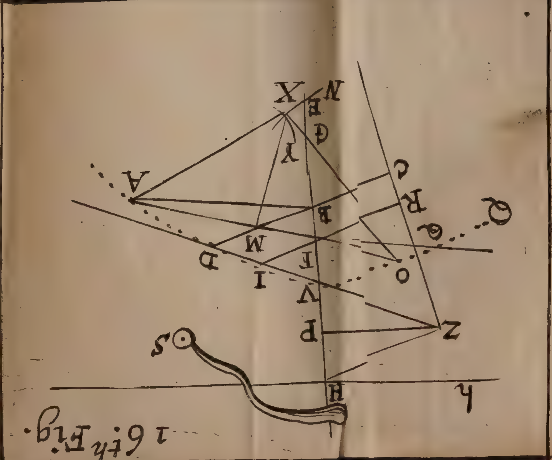

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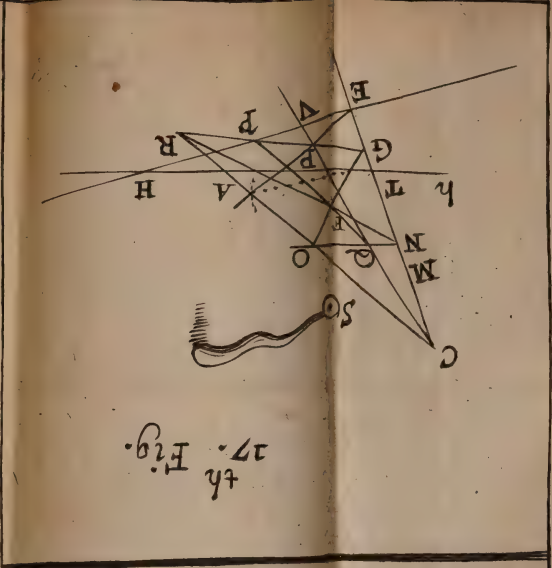
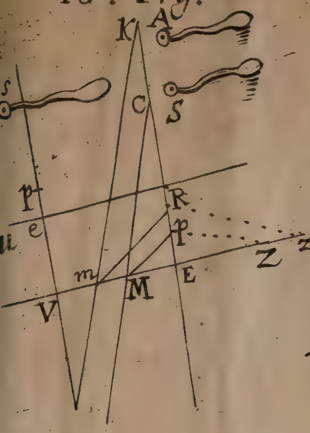


17th Fig.

18th. Fig.



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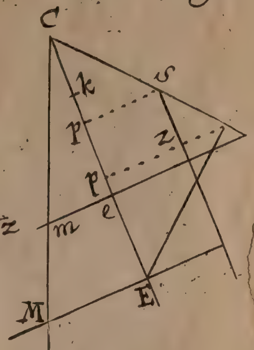

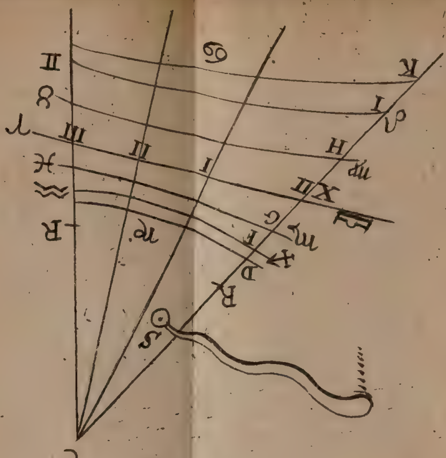

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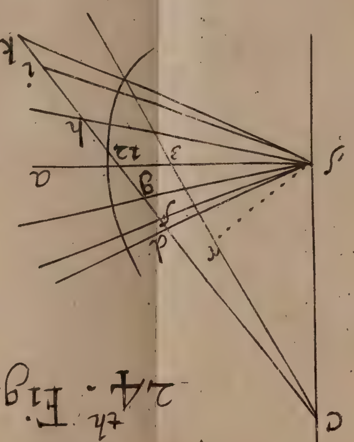

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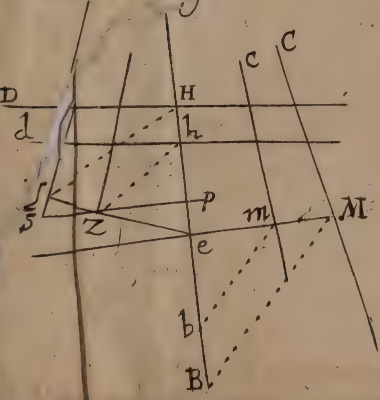

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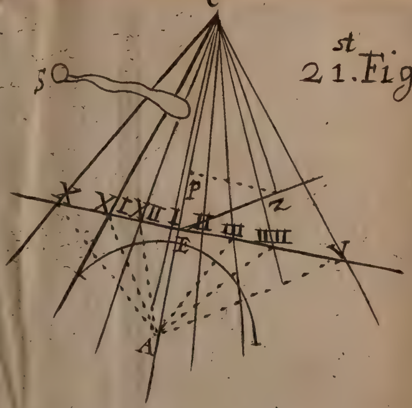
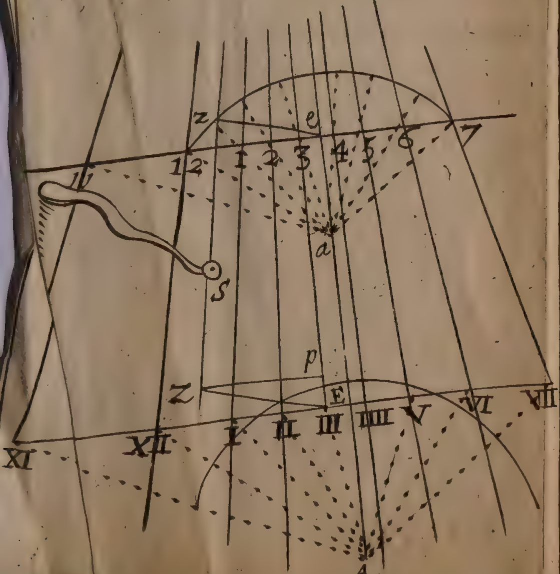
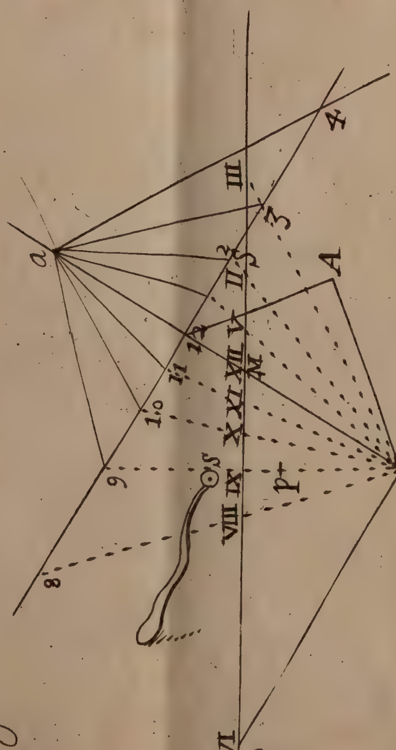




22^{d}. Fig.



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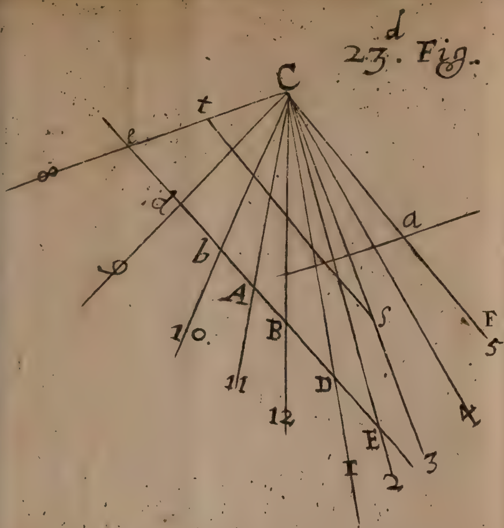

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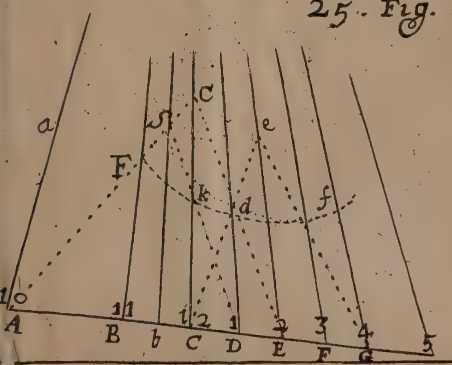

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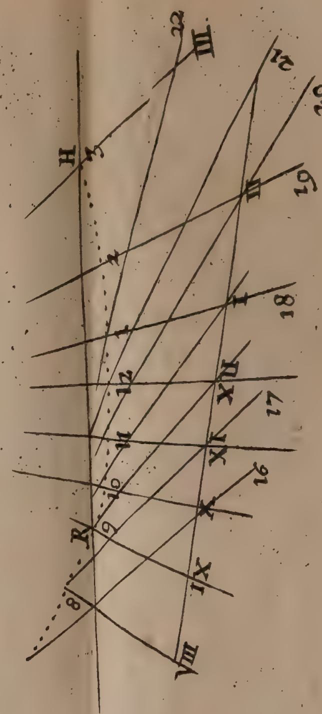

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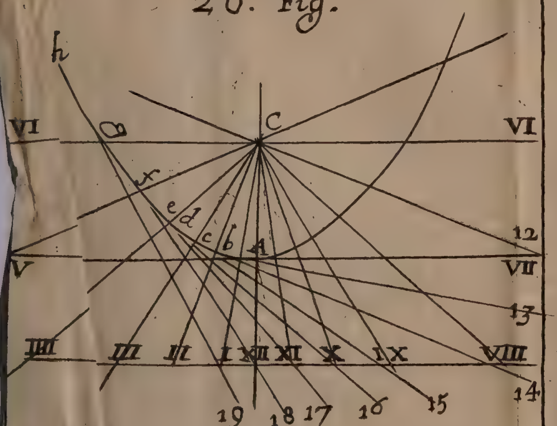

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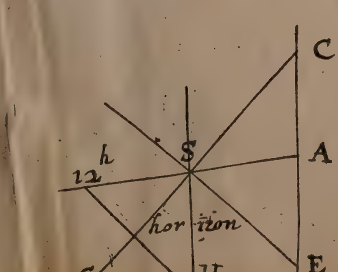

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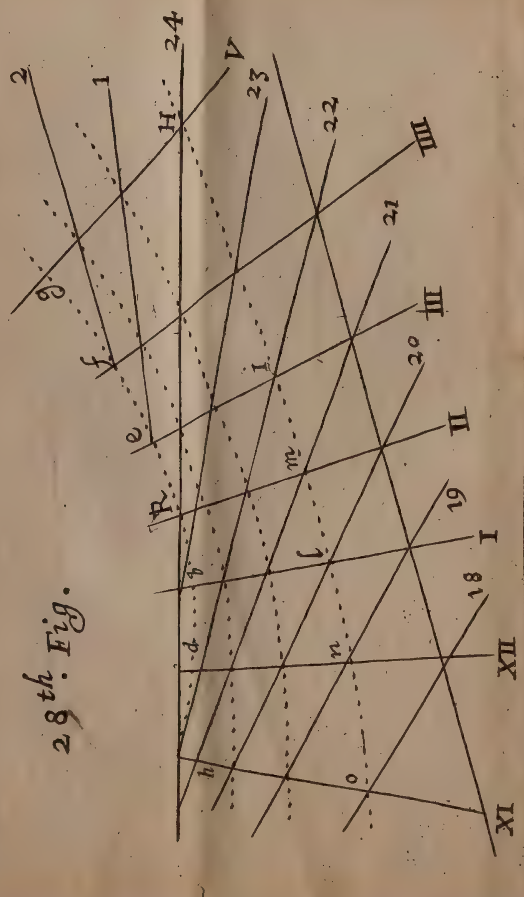

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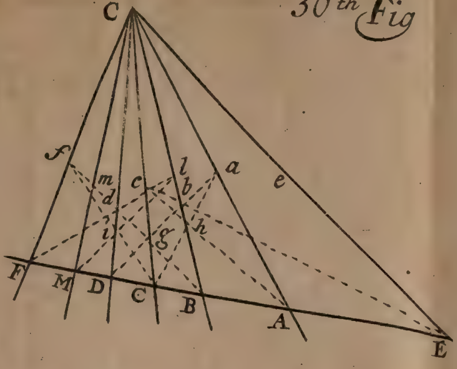

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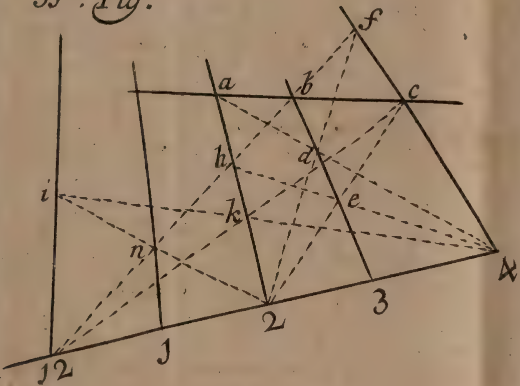

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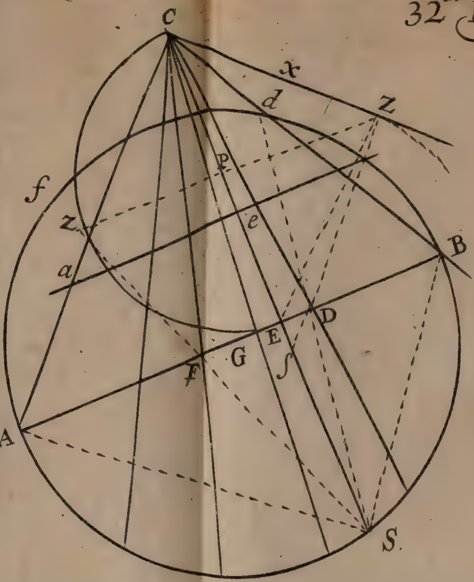

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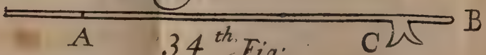

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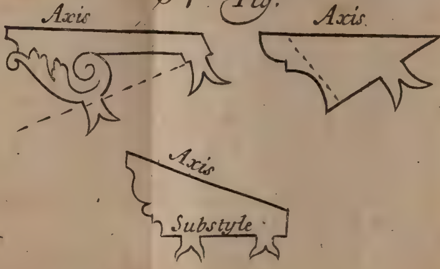
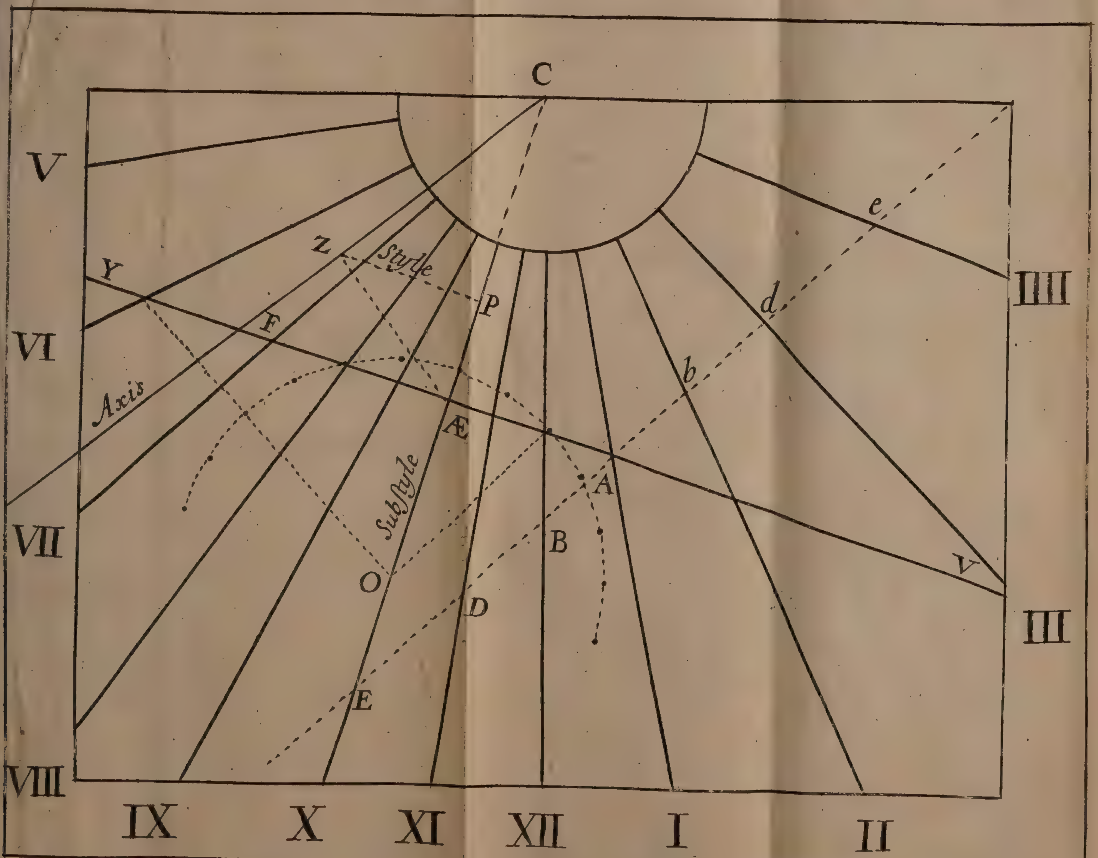


35 Figure



THis Geometrical way of Mr. *De la Hire* for Drawing *Sun-Dials* on *Fixed Planes* is universal. The *Instruments* that are used to perform the *Practice* thereof are only a *Plain Scale* and a *Pair of Compasses* for finding the *Substyler* and *Equinoctial Lines* on the given *Plane*, with the situation of the *Axis* of the *Style* in respect of the said *Plane*, the Observations being made by the *Sun* from several *Shadows* of the *Point* of a *Fixed Style* set upon the said *Plane*. And by a *Level* and *Plumb Line* for finding the situation of the *Meridian* and *Horizontal Lines* of the *Place* on the *Plane* of the *Dial*; which *Practice* of *Dialling* he chiefly demonstrates from the Intersections of *Planes*, the Doctrine whereof is contained in the 11th Book of *Euclid's Elements*. And because he has given no particular Example of this general way of *Dialling* in this *Treatise*, I have thought good to add this Example following:

A particular Example of this General Method of *Dialling* Illustrated by Numbers.

There is a *Style* fix'd on a *Fixed Plane*, whose perpendicular height is S P 1 inch 64 parts, or 164 parts. Now, the *Sun* shining, if there be taken Two Points of Shadow of the top of the *Style* S on the said *Plane*, as A and B, on that day when the *Suns* Declination is 21 deg. 40 min. South, and that P B the Distance of the Point B,

from P the foot of the *Style* may be 55 parts, and the Diſtance of the Point A from P may be PA 366 parts, and that the Diſtance of the Point A from B may be BA 415 parts.

Now from theſe things given, it is required to find the ſituation of the *Subſtylar* and *Equinoctial Lines*, and to find the *Center* of the *Dial* on the ſaid *Plane*, and to determine the poſition of the *Axis* of the *Dial* by *Calculation.*

[The Geometrical Operation of this is to be found in the Sixth Chapter, page 16th of this *Treatiſe*, and in the 5th Figure.]

Firſt in the Right-angled Triangle SPB.

There is given PS 164 parts, the perpendicular height of the Style, and PB 55 parts the diſtance of the Point of Shadow B from P the foot of the Style;

To find the Vertical Angle PSB, and the Hypotenuſe SB, which is the length of the Ray of the Sun from the top of the Style S to the Point of Shadow B,

The Proportion is as SP 164 parts,
Is to PB 55 parts:
So is the Radius,
To the Tang. of the Angle PSB 18 d. 32 m.

And

And as the Sine of the Angle PSB 18 d. 32 min.
Is to PB 55 parts,
So is the Sine of 90 deg.
To SB 173 parts.

Secondly, In the Right-angled Triangle SPA,
There is given the perpendicular PS 164 parts
And the Base PA 366 parts,
To find the *Vertical Angle* PSA and the *Hypotenuse* SA.
The Proportion is, As SP 164 parts.
Is to PA 366 parts.
So is the Radius
To the Tangent of the Angle PSA 65 d. 52 m.

And as the Sine of the Angle PSA 65 d. 52 m.
Is to PA 366 parts,
So is the Sine of 90 degrees,
To SA 401 parts.

Thirdly in the Obliquangular Triangle *fdb.*

There is given the two sides *fb* 173 parts and *fd* 100 parts, taken at pleasure, and the contained Angle *dfb* 68 deg. 20 min. the Complement of the Suns Declination.

To find the other two Angles *ſdb* and *ſbd*, and the third ſide *db*.

From ————————	180 deg.	00 min.
Take the given Angle *dſb*	68	20
And there ſhall remain ———	111 deg.	40 min.
The half whereof is ———	55 deg.	50 min.

Then the greater ſide is *ſb* 173 parts,
And the leſs ſide is *ſd* 100 parts.

Then as the Sum of the ſides 273 parts,
Is to the difference of the ſides 73 parts:
So is the Tangent of 55 deg. 50 min.
To the Tangent of 21 deg. 30 min.

The Sum is *ſdb* 77 deg. 20 min.
The Difference is *ſbd* 34 deg. 20 min.

Then as the Sine of the Angle *ſbd* 34 d. 20 m.
Is to the Sine of the Angle *dſb* 68 d. 20 m.
So is *ſd* 100 parts,
To *db* 164 parts.

Fourthly, In the Obliquangular Triangle *ſda*.

There is given the two ſides *ſa* 401 parts, and *ſd* 100 parts, and the contained Angle *dſa*,

To

To find the other two Angles *ſd a* and *ſa d*, and the third ſide *da*.

Then the greater ſide is *ſa* 401 parts,
And the leſs ſide is — *ſd* 100

Then as the Sum of the ſides 501
Is to the Difference of the ſides 301
So is the Tangent of 55 deg. 50 min.
To the Tangent of 41 31

The Angle *ſda* —— 97 21
The Angle *ſad* —— 14 19

Then as the Sine of the Angle *ſad* 14 d. 19 m.
Is to the Sine of the Angle *aſd* 68 20
So is *ſd* 100 parts,
To *da* 376 parts.

Fifthly, In the Obliquangular Triangle BAT.

There is given the Baſe B A 415 parts, and the two ſides A T 376 parts, and B T 164 parts, To find the Segments of the Baſe O A and O B, and the perpendicular O T.

The greater ſide is A T 376 parts,
The leſs ſide is B T 164 parts.
The Sum of the ſides — 540 parts
The Difference of the ſides 212

Therefore as the Baſe B A —— 415 parts,
Is to the Sum of the ſides —— 540
So is the difference of the ſides —— 212
To the difference of the Seg. of the Baſe 276

Therefore to B A —— 415 parts
Add —————— 276

The Sum is 691

The half Sum is A O —— $345\frac{1}{2}$.

And from B A 415 parts,
Subſtract —— 276

The Difference is 139

The half is B O — $69\frac{1}{2}$.

The ſide B T is 164 parts,
And B O is —— 69 $\frac{1}{2}$

The Sum is 233 $\frac{1}{2}$ parts, Log. 2.367356
The Difference is 95 parts, Log. 1.977724

Sum Log. 4.345080

Half Sum Log. 2.172540

Log. of O T 149 parts.

Sixthly, In the Obliquangular Triangle BPA,

There is given the Baſe B A 415 parts, and the two ſides P A 366 parts and P B 55 parts,
To find B x and A x the Segments of the Baſe and the perpendicular P x.

The greater ſide P A 366 parts,
The leſs ſide P B —— 55

Their Sum is —— 421
Their Difference is 311

Therefore as the Baſe B A 415 parts,
Is to the Sum of the ſides 421 parts:
So is the difference of the ſides 311
To —————————— 315

Therefore to B A ———— 415 parts
Add ———— 315

The Sum is ———— 730

The ½ is the greater Segment A x. 365

And from B A ———— 415 parts
Substract ———— 315

The Difference is ———— 100

The ½ is the less Segment B x —— 50

The less side B P 55
The Segment B x 50

The Sum is — 105 Log. 2.021189
The difference is 5 Log. 0.698970

The Sum of the Log. ———— 2.720159

The ½ Sum is the Log. of P x 23, 2.360079½

B O 69 parts
B x 50

O x 19

Seventh-

Seventhly, In the Right-angled Triangle P G I,

There is given the Hypotenuse PI 100 parts, and the side P G 19 parts,

To find the side GI.

The greater side PI 100 parts
The less side P G 19

Their Sum —— 119 parts, Log. 2.075547
Their difference 81 parts, Log. 1.903485

The Sum of the Log. 3.984032

The ½ Sum is the Log. of GI 98 parts, 1.992016

Eightly, In the Right-angled Triangle K G O,

There is given KG 164 parts and O G 23 parts,
To find the Angle G K G, and the side O K.

The Proportion is as G K 164 parts,
Is to G O 23 parts:
So is the Radius,
To the Tangent of the Angle G K O 7 d. 59 m.

And as the Sine of 7 deg. 59 min.
Is to G O 23 parts:
So is the Radius,
To O K 166 parts.

Ninthly, In the Obliquangular Triangle OKD,

There is given the Base O K 166 parts, and O D 149 parts, equal to the Radius of the Semicircle L D T and K D 98 parts,

To find the Angle O K D.

—— O D 149 parts
K D 98 parts

Their Sum 247
Their Difference 51

Then as the Base O K 166 parts
Is to the Sum ——— 247
So is the difference—— 51
To ————— 76

Therefore

Therefore to OK 166 parts
Add 76

The Sum is 242

The $\frac{1}{2}$ Sum is O y 121

And from O K 166 parts
Subſtract ——— —— 76

The difference is —— 90

The $\frac{1}{2}$ difference is K y 45

Then as D K 98
Is to K y 45
So is the Sine of 90 degrees,
To the Sine of the Angle K D y 27 d. 20 m.
The Complement is the Angle DKy 62 40
Subtract the Angle G K O —— 7 59

Remains the Angle D K z ——— 54 41.

Tenthly,

Tenthly, In the Right-angled Triangle D K z,

There is given the Hypotenuſe DK 98 parts and the Angle D K z 54 degrees 41 minutes,

To find the perpendicular D z and the Baſe K z:

For as the Sine of D z K 90 deg.
Is to the Hypotenuſe D K 98 parts
So is the Sine of the Angle D K z 54 d. 41 m.
To the perpendicular D z 57 parts.

Eleventhly, In the Right-angled Triangle P G Q,

There is given the Baſe P G 19 parts,
And the perpendicular G Q 80 parts,
To find the Hypotenuſe P Q.
The proportion is as G Q 80 parts,
Is to P G 19 parts:
So is the Radius,
To the Tangent of the Angle P G Q 13 deg. 20 min.

And

And as the Sine of the Angle P Q G 13 d. 20 m.
Is to P G 19 parts:
So is the Sine of 90 degrees,
To P Q 82 parts.

Then from N P 164
Take —— N *r* 57

Remains—— *r* P 107

Twelfthly, To find P C;

There is given N *r* 57, and *r* M 82, and N P 164. The Proportion is,

As N *r* 57
Is to *r* M 82
So is NP 164
To P C — 236

Lastly,

Laftly, To find P E,

The Proportion is, As P C 236
Is to P N 164
So is P N 164
To P E 114

Now if the Plane of the Dial be on an upright Wall, the Meridian Line may be found as in the 13th Chapter of the Firft Part, Figure 14th, and the Hours may be fet on the Equinoctial Line by the Firft Chapter of the Second Part, Fig. 21.

And Six Hours being given, the reft may be found by the Third Chapter of the Second Part.

Or Four Hours and the Equinoctial Line being given, the reft may be

be found by the Ninth Chapter of the Second Part.

And then the Dial may be drawn as in the 35th Figure.

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FINIS.